

MAT.05.PT.4.STORG.A.413 Claim 4

Sample Item ID:	MAT.05.PT.4.STORG.A.413
Title:	Storage Facility
Grade:	05
Primary Claim:	Claim 4: Modeling and Data Analysis Students can analyze complex, real-world scenarios and can construct and use mathematical models to interpret and solve problems.
Secondary Claim(s):	Claim 1: Conceptual Understanding and Procedural Fluency Students can explain and apply mathematical concepts and interpret and carry out mathematical procedures with precision and fluency.
Primary Content Domain	Measurement and Data
Secondary Content Domain(s):	Operations and Algebraic Thinking Number and Operations in Base Ten Geometry
Assessment Target(s):	4 A: Apply mathematics to solve problems arising in everyday life, society, and the workplace. 4 B: Construct, autonomously, chains of reasoning to justify mathematical models used, interpretations made, and solutions proposed for a complex problem. 4 D: Interpret results in the context of a situation. 4 E: Analyze the adequacy of and make improvements to an existing model or develop a mathematical model of a real phenomenon. 1 D: Perform operations with multi-digit whole numbers and with decimals to hundredths. 1 I: Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.
Standard(s):	5.OA.3, 5.NBT.7, 5.MD.3, 5.MD.5, 4.OA.5, 4.MD.3, 3.MD.6, 3.MD.7
Mathematical Practice(s):	1, 2, 4
DOK:	3
Item Type:	PT
Score Points:	17
Difficulty:	H
How this task addresses the "sufficient evidence" for this claim:	The student uses problem-solving strategies to organize the area of rectangles within a given amount of space. The student uses numeric operations to find the volume of rectangular prisms, the monthly cost, and the average cost per unit. The student justifies why the original mathematical model is insufficient and makes improvements given the original data. Finally, the student uses problem-solving strategies based upon the new data he/she created to answer various mathematical concepts.

Target-specific attributes (e.g., accessibility issues):	Accommodations may be necessary for students with fine- motor-skill challenges and language-processing challenges.
Stimulus/Source:	http://www.publicstorage.com
Notes:	Multi-part task
Task Overview:	Students must perform various calculations in order to find the lowest cost for a specified amount of volume using fixed storage space dimensions.
Teacher preparation / Resource requirements:	None
Teacher Responsibilities During Administration:	Monitor individual student work
Time Requirements:	Two sessions totaling no more than 120 minutes. <i>Part A</i> and <i>Part B</i> should be completed in Session 1. <i>Parts C</i> through <i>Part E</i> should be completed in Session 2.

Prework: (May be blank)

Storage Facility

The owner of an empty building wants to construct walls to divide the building into individual storage units. All the space in the building will be used for the storage units. Each storage unit will have a door on the outside (perimeter) of the building.

Part A

Design the Storage Building

The grid that follows represents the rectangular floor of the building. Draw lines on the grid to divide the floor into 9 individual storage units.

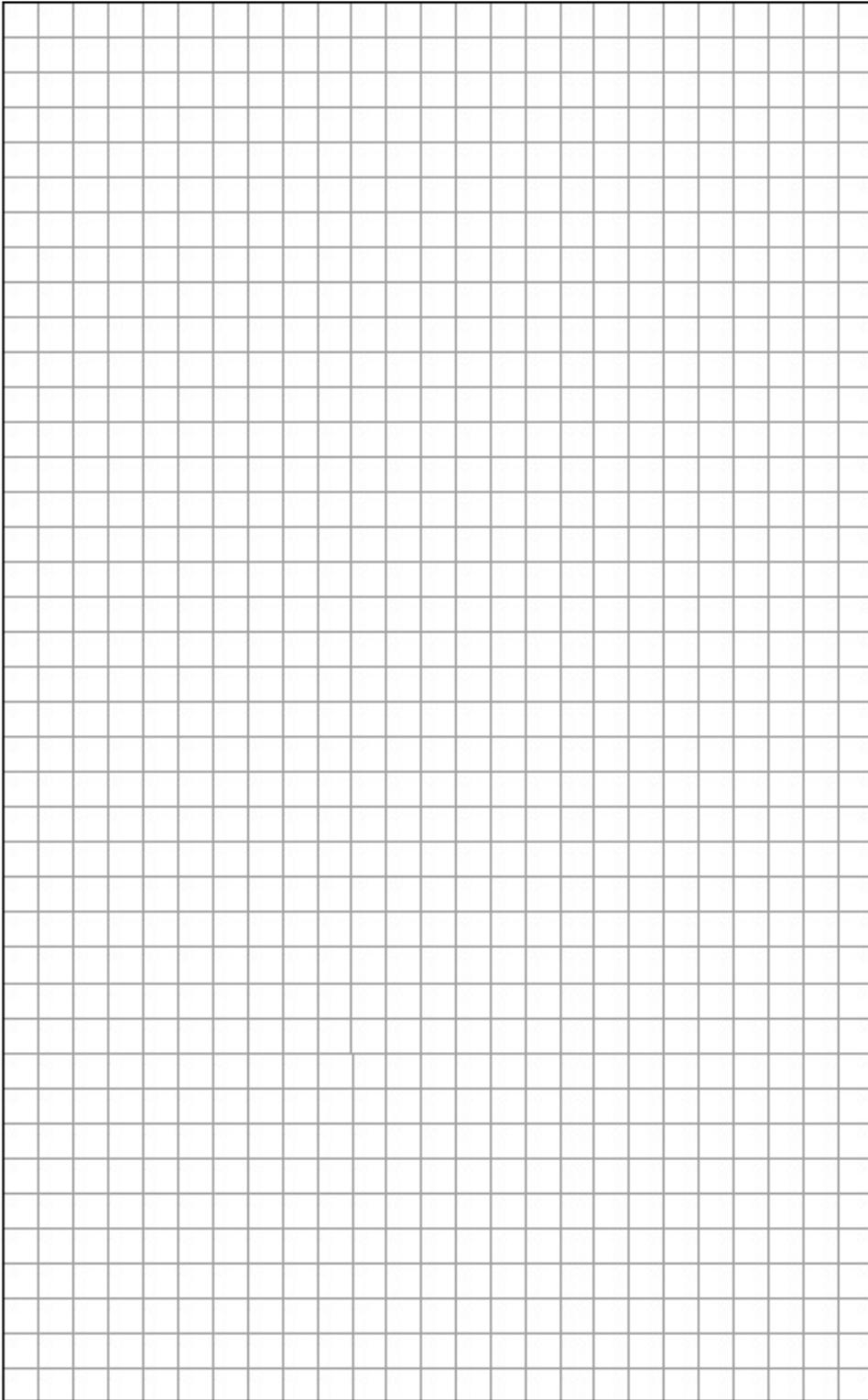
The following rules apply:

- Each unit must have a rectangular floor.
- Each unit will share one side with the outside edge (perimeter) of the building.
- The storage units must not overlap.

The sizes of the storage units and the number of each size that must be included in the drawing are shown in the following table.

Dimensions of Floor (in feet)	Number of Storage Units in Building
5 by 5	1
5 by 10	2
5 by 15	1
10 by 10	2
10 by 15	1
10 by 20	1
10 by 25	1

[Note: Instructions for creating lines using tools on the computer will be provided as necessary.]



SCALE
1 square = 1 foot

Part B**Determine the Monthly Rent**

The building owner needs to determine the amount of monthly rent he will charge his customers for each individual storage unit. He wants to charge a fixed amount for each cubic foot of volume in the storage units.

Complete the table below to find the volume of each storage unit and the cost of monthly rent for each unit based on the volume.

Dimensions of Floor (in feet)	Height of Storage Unit (in feet)	Volume of Storage Unit (in cubic feet)	Monthly Rent
5 by 5	10	250	\$30
5 by 10	10		
5 by 15	10		
10 by 10	10	1000	\$120
10 by 15	10		
10 by 20	10		
10 by 25	10		

You will not be able to return to *Parts A* and *B* after completing the table and clicking “submit.”

End of Session 1

Session 2**Part C****Recalculating the Monthly Rents**

The completed table from *Part B* is shown below.

Dimensions of Floor (in feet)	Height of Storage Unit (in feet)	Volume of Storage Unit (in cubic feet)	Monthly Rent
5 by 5	10	250	\$30
5 by 10	10	500	\$60
5 by 15	10	750	\$90
10 by 10	10	1000	\$120
10 by 15	10	1500	\$180
10 by 20	10	2000	\$240
10 by 25	10	2500	\$300

The building owner determined that some of the monthly rents in *Part B* are not reasonable. He wants the cost for each cubic foot of volume to decrease as the volume of the storage unit increases.

In the table below, the owner decided to charge \$0.12 for each cubic foot of volume in **only** the smallest storage unit.

- Complete the table by determining a cost for each cubic foot of volume and a corresponding monthly rent for each individual storage unit.
- Make sure the **cost** for each cubic foot **decreases** as the volume of the storage unit increases.
- Make sure the monthly **rent increases** as the **volume** of the storage unit **increases**.

Dimensions of Floor (in feet)	Volume of Storage Unit (in cubic feet)	Cost for each Cubic Foot of Volume	Monthly Rent
5 by 5	250	\$0.12	\$30
5 by 10	500		
5 by 15	750		
10 by 10	1000		
10 by 15	1500		
10 by 20	2000		
10 by 25	2500		

Part D**Total Monthly Rent Collected**

The table of storage unit sizes from *Part A* is shown below.

Dimensions of Floor (in feet)	Number of Storage Units in Building
5 by 5	1
5 by 10	2
5 by 15	1
10 by 10	2
10 by 15	1
10 by 20	1
10 by 25	1

What is the total amount of monthly rent, in dollars, the owner should collect using the table shown at the beginning of Session 2 (*Part B* completed table)?

[Student is able to view the *Part B* completed table at the beginning of Session 2.]

\$

Use mathematics to justify your answer.

What is the total amount of monthly rent, in dollars, the owner should collect using the **adjusted** rent amounts from your completed table in *Part C*? Use mathematics to justify your answer.

[Student is able to view the completed table in *Part C*]

\$

Identify one reason why the owner might choose to use your monthly rent table from *Part C* instead of the monthly rent table from *Part B*.

Part E

Customer Advice

Ms. Prater is the first customer to rent storage units in the building. She needs a total of 4000 cubic feet of space. In the space below, make a recommendation to Ms. Prater about which storage units she should rent. Use the table from *Part C* to calculate the total rent on the storage units you chose.

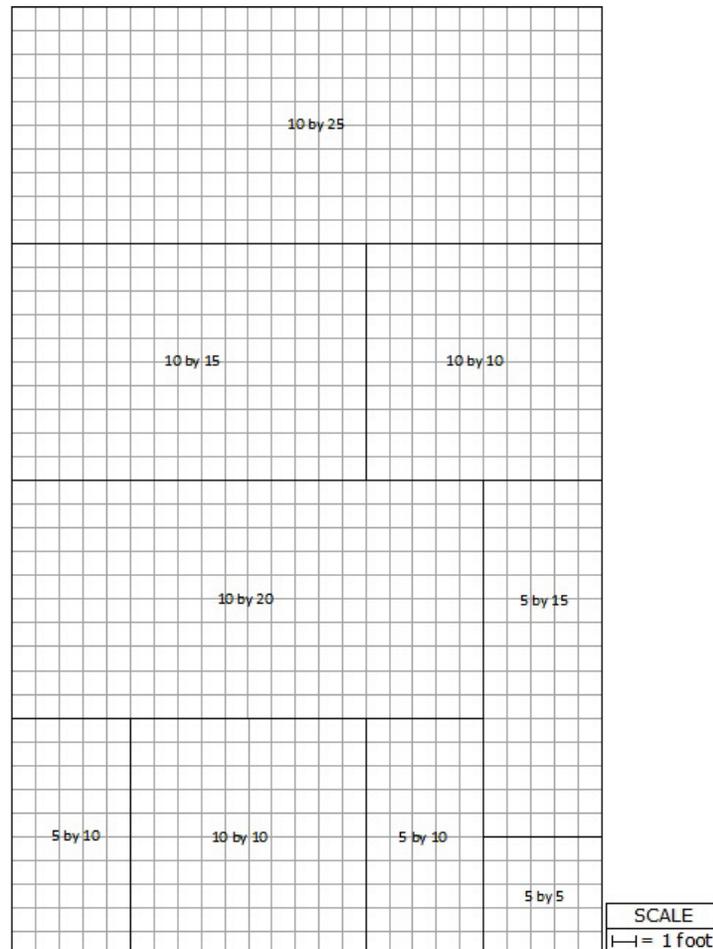
Justify why you chose the combination of storage units that you did using mathematics and/or words.

End of Session 2

Sample Top-Score Response:

Part A

The grid is divided into correctly sized and labeled rectangles. None of the rectangles overlap, and every rectangle shares at least one side with the outside edge of the grid. A possible response is shown below.



Part B

The student completes the table as shown.

Dimensions of Floor (in feet)	Height of Storage Unit (in feet)	Volume of Storage Unit (in cubic feet)	Monthly Rent
5 by 5	10	250	\$30
5 by 10	10	500	\$60
5 by 15	10	750	\$90
10 by 10	10	1000	\$120
10 by 15	10	1500	\$180
10 by 20	10	2000	\$240
10 by 25	10	2500	\$300

Part C

The student completes the table. The entries in the “Monthly Rent” column should **increase** from top to bottom and the “Cost for Each Cubic Foot of Volume” entries should **decrease** from top to bottom. A possible correct response is shown.

Dimensions of Floor (in feet)	Volume of Storage Unit (in cubic feet)	Cost for Each Cubic Foot of Volume	Monthly Rent
5 by 5	250	\$0.12	\$30
5 by 10	500	\$0.11	\$55
5 by 15	750	\$0.10	\$75
10 by 10	1000	\$0.09	\$90
10 by 15	1500	\$0.08	\$120
10 by 20	2000	\$0.07	\$140
10 by 25	2500	\$0.06	\$150

Part D

The student writes \$1200 and shows work needed to arrive at this answer.

The student correctly computes the total rent collected for the table he/she created in *Part C*. The total should be the sum of the monthly rents plus the rent amounts in the second and fourth rows. For the sample table shown above, this total would be \$805.

The student identifies and explains one reason why the building owner would choose the table in *Part C*, such as, “The rent costs less for most of the storage units so it might be easier to rent them to customers.” **OR** “The customers who need more space will pay less for each cubic foot of volume.”

Part E

The student chooses a combination of storage units that have a total volume of 4000 cubic feet. Possible combinations are shown below.

2500 cubic feet + 1500 cubic feet
 2500 cubic feet + 1000 cubic feet + 500 cubic feet
 2500 cubic feet + 750 cubic feet + 500 cubic feet + 250 cubic feet
 2000 cubic feet + 1500 cubic feet + 500 cubic feet
 2000 cubic feet + 2 x 1000 cubic feet
 2000 cubic feet + 1000 cubic feet + 750 cubic feet + 250 cubic feet
 2000 cubic feet + 1000 cubic feet + 2 x 500 cubic feet
 2000 cubic feet + 750 cubic feet + 2 x 500 cubic feet + 250 cubic feet
 1500 cubic feet + 2 x 1000 cubic feet + 500 cubic feet
 1500 cubic feet + 1000 cubic feet + 750 cubic feet + 500 cubic feet + 250 cubic feet
 2 x 1000 cubic feet + 750 cubic feet + 2 x 500 cubic feet + 250 cubic feet

The student should calculate the correct total rent based on their completed table from *Part D*.

The student should justify that the combination he/she chose results in the lowest total monthly rent. (Note: The student does not need to calculate the total monthly rent for each

combination above). In most cases, the lowest monthly rent should result from the combination of the 2500 cubic foot storage unit and the 1500 cubic foot storage unit.

Scoring Notes:

Each part of the task is evaluated individually. The total number of points is determined by adding the points assigned for each part.

Scoring Rubric:

Responses to this item will receive 0-17 points, based on the following:

Part A

4 points: Thorough understanding of rectangle area and dimensions. Thorough understanding of the given directions. The student correctly divided the grid into 9 non-overlapping rectangles of the appropriate sizes. The student correctly labeled each rectangle with its dimensions. The student divided the grid so that each smaller rectangle shares at least one side with the outer edge of the grid.

3 points: Partial understanding of rectangle area and dimensions. Partial understanding of the given directions. The student correctly divided the grid into 9 rectangles of the appropriate sizes. The student labeled each rectangle with its dimensions. The student may not have divided the grid so that each smaller rectangle shares at least one side with the outer edge of the grid.

2 points: Partial understanding of rectangle area and dimensions. Partial understanding of the given directions. The student divided the grid into 9 rectangles of the appropriate sizes. The student labeled each rectangle with its dimensions. The rectangles may not completely fill the grid or may not represent the sizes given in the directions. The student may not have divided the grid so that each smaller rectangle shares at least one side with the outer edge of the grid.

1 point: Limited understanding of rectangle area and dimensions. Limited understanding of the given directions. The student divided the grid into fewer than 9 rectangles of the appropriate sizes. The student correctly labeled each rectangle with its dimensions. Some of the rectangles may be overlapping. The student may not have divided the grid so that each smaller rectangle shares at least one side with the outer edge of the grid.

0 points: No understanding of rectangle area and dimensions. No understanding of the given directions. The student may have not divided the grid or divided the grid into nonrectangular sections or rectangular sections of incorrect sizes.

Part B

3 points: Thorough understanding of calculating the volume of a rectangular prism. Thorough understanding of calculating the monthly rent based on a fixed cost for each cubic foot of space. The student correctly filled in at least 8 of the cells in the table (may have calculated one volume incorrectly and the associated monthly rent incorrectly).

2 points: Thorough understanding of calculating the volume of a rectangular prism. Limited

understanding of calculating the monthly rent based on a fixed cost for each cubic foot of space. The student correctly filled in at least 4 of the cells in the first column of the table.

OR Limited understanding of calculating the volume of a rectangular prism. Thorough understanding of calculating the monthly rent based on a fixed cost for each cubic foot of space. The student correctly filled in at least 4 of the cells in the second column of the table based on the incorrect values in the first column and the given monthly rents.

1 point: Partial understanding of calculating the volume of a rectangular prism. Limited understanding of calculating the monthly rent based on a fixed cost for each cubic foot of space. The student correctly filled in at least 3 of the missing volumes in the table but did not correctly calculate the associated monthly rents. **OR** Limited understanding of calculating the volume of a rectangular prism. Partial understanding of calculating the monthly rent based on a fixed cost for each cubic foot of space. The student incorrectly filled in at least 3 of the missing volumes in the table but correctly calculated the associated monthly rents.

0 points: No understanding of calculating the volume of a rectangular prism. No understanding of calculating the monthly rent based on a fixed cost for each cubic foot of space.

Part C

3 points: Thorough understanding of multiplying a decimal by a whole number to calculate the monthly rent for each storage unit size. Thorough understanding of selecting values that demonstrate increasing and decreasing trends as given in the directions. The student correctly provided values in the third column that decrease from top to bottom and used them to calculate correctly the values in the fourth column, which must increase from top to bottom.

2 points: Thorough understanding of multiplying a decimal by a whole number to calculate the monthly rent for each storage unit size. Partial understanding of selecting values that demonstrate increasing and decreasing trends as given in the directions. The student provided values in the third column that do not decrease from top to bottom, but used them to calculate correctly the values in the fourth column.

1 point: Limited understanding of multiplying a decimal by a whole number to calculate the monthly rent for each storage unit size. Limited understanding of selecting values that demonstrate increasing and decreasing trends as given in the directions. The student provided some values in the third column that decrease from top to bottom. The student calculated at least one value in the fourth column based on the associated value chosen for the third column. **OR** Partial understanding of multiplying a decimal by a whole number to calculate the monthly rent for each storage unit size. Thorough understanding of selecting values that demonstrate increasing and decreasing trends as given in the directions. The student correctly provided values in the third column that decrease from top to bottom, but did not use them to calculate correctly the values in the fourth column.

0 points: No understanding of multiplying a decimal by a whole number to calculate the monthly rent for each storage unit size. No understanding of selecting values that demonstrate increasing and decreasing trends as given in the directions.

Part D

4 points: Thorough understanding of combining information from different tables to calculate the total monthly rent for the building. Thorough understanding of the preference for a rent table not based on a fixed cost for each cubic foot of space. The student provided a correct total amount of monthly rent for *Part B* and *Part C* (based upon the values calculated by the student in the fourth column of the table in *Part C*). The student explains why a rent table that is not based on a fixed cost for each cubic foot of space is preferable.

3 points: Partial understanding of combining information from different tables to calculate the total monthly rent for the building. Thorough understanding of the preference for a rent table not based on a fixed cost for each cubic foot of space. The student provided an incorrect total amount of monthly rent for *Part B* and *Part C* (based upon the values calculated by the student in the fourth column of the table in *Part C*) by not including the two storage unit sizes that are represented twice. **OR** The student provided a correct total amount of monthly rent for *Part B* or *Part C*. The student explains why a rent table that is not based on a fixed cost for each cubic foot of space is preferable.

2 points: Partial understanding of combining information from different tables to calculate the total monthly rent for the building. Understanding of the preference for a rent table not based on a fixed cost for each cubic foot of space is not adequately demonstrated. The student may have provided an incorrect total amount of monthly rent for *Part B* or *Part C* (based upon the values calculated by the student in the fourth column of the table in *Part C*). The student provides a reason why the owner might choose the monthly rent table from *Part C*, but the reason may not be convincing.

1 point: Limited understanding of combining information from different tables to calculate the total monthly rent for the building. Limited understanding of the preference for a rent table not based on a fixed cost for each cubic foot of space. The student provided an incorrect total amount of monthly rent for only one table in *Part B* or *Part C* (based upon the values calculated by the student in the fourth column of the table in *Part C*) by not including the two storage unit sizes that are represented twice. The student provides a reason why the owner might choose the monthly rent table from *Part C*, but the reason may not be convincing.

0 points: No understanding of combining information from different tables to calculate the total monthly rent for the building. No understanding of the preference for a rent table not based on a fixed cost for each cubic foot of space. The student provided an incorrect total amount of monthly rent for both tables in *Part B* and *Part C*. The student may provide a reason why the owner might choose the monthly rent table from *Part C*, but the reason is flawed.

Part E

3 points: Thorough understanding of adding volumes. Thorough understanding that storage units with greater volumes cost less for each cubic foot of space. The student selected two or more storage units that have a combined volume of exactly 4000 cubic feet. The student shows work to indicate that more than one combination of storage units was considered, but the combination chosen was the one that results in the lowest total monthly rent (in most cases, this should be the 2500-square-foot and 1500-square-foot storage units). The student justifies the selection by explaining that the storage units with greater volumes cost less for each cubic foot of space.

2 points: Thorough understanding of adding volumes. Partial understanding that storage units with greater volumes cost less for each cubic foot of space. The student selected two or more storage units that have a combined volume of exactly 4000 cubic feet. The student shows work to indicate that more than one combination of storage units was considered, but the combination chosen was one that results in one of the lowest total monthly rent. The student does not justify the selection.

1 point: Partial understanding of adding volumes. Limited understanding that storage units with greater volumes cost less for each cubic foot of space. The student selected two or more storage units that have a combined volume of exactly 4000 cubic feet. The combination chosen does not result in the lowest total monthly rent. The student does not justify the selection.

0 points: No understanding of adding volumes. No understanding that storage units with greater volumes cost less for each cubic foot of space. The student selected two or more storage units that do not have a combined volume of exactly 4000 cubic feet. The student does not justify the selection.