

Grade 8 Mathematics Item Specification C1 TG

<p>Claim 1: Concepts and Procedures Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.</p>	
<p>Content Domain: Geometry</p>	
<p>Target G [m]: Understand congruence and similarity using physical models, transparencies, or geometry software. (DOK Levels 1, 2)</p> <p>Technology enhanced items will be used to allow students to “draw” lines, line segments, angles, and parallel lines after undergoing rotations, reflections, and translations. Similar technology enhanced items will ask students to produce a new figure or part of a figure after undergoing dilations, translations, rotations, and/or reflections.</p> <p>Other tasks will present students with two figures and ask students to describe a series of rotations, reflections, translations, and/or dilations to show that the figures are similar, congruent, or neither. Many of these tasks will contribute evidence for Claim 3, asking students to justify reasoning or give a critique of a sample reasoning.</p>	
<p>Standards: 8.G.A, 8.G.A.1, 8.G.A.2, 8.G.A.3, 8.G.A.4, 8.G.A.5</p>	<p>8.G.A Understand congruence and similarity using physical models, transparencies, or geometry software</p> <p>8.G.A.1 Verify experimentally the properties of rotations, reflections, and translations:</p> <ol style="list-style-type: none"> a. Lines are taken to lines, and line segments to line segments of the same length. b. Angles are taken to angles of the same measure. c. Parallel lines are taken to parallel lines. <p>8.G.A.2 Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.</p> <p>8.G.A.3 Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.</p> <p>8.G.A.4 Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.</p> <p>8.G.A.5 Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. <i>For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and explain, in terms of transversals why this is so.</i></p>
<p>Related Below-Grade and Above-Grade Standards for Purposes of Planning for Vertical Scaling:</p> <p>7.G.A, 7.G.A.1, 7.G.2, 7.G.A.3, 7.G.B, 7.G.B.4, 7.G.B.5,</p>	<p>Related Grade 7 Standards</p> <p>7.G.A Draw, construct, and describe geometrical figures and describe the relationship between them.</p> <p>7.G.A.1 Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.</p> <p>7.G.A.2 Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than</p>

Grade 8 Mathematics Item Specification C1 TG

<p>G-CO.A, G-CO.A.1, G-CO.A.2, G-CO.A.3, G-CO.A.4, G-CO.A.5</p>	<p>one triangle, or no triangle. 7.G.A.3 Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids. 7.G.B Solve real-life and mathematical problems involving angle measure, area, surface area, and volume. 7.G.B.4 Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle. 7.G.B.5 Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.</p> <p>Related High School Standards</p> <p>G-CO.A Experiment with transformations in the plane. G-CO.A.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc. G-CO.A.2 Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch). G-CO.A.3 Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself. G-CO.A.4 Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments. G-CO.A.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.</p>
<p>DOK Levels:</p>	<p>1, 2</p>
<p>Achievement Level Descriptors:</p>	
<p>RANGE Achievement Level Descriptor (Range ALD)</p> <p>Target G: Understand congruence and similarity using physical models, transparencies, or geometry software.</p>	<p>Level 1 Students should be able to identify reflections, rotations, and translations and show the result of these rigid motions on figures.</p> <p>Level 2 Students should be able to construct reflections and translations of figures in a coordinate plane and identify dilations and the results of dilations on figures.</p> <p>Level 3 Students should be able to understand and describe the impact of a transformation on a figure and its component parts with or without coordinates. They should be able to use or describe a sequence of transformations to determine or exhibit the congruence of two figures. They should also be able to construct rotations and dilations of figures in a coordinate plane.</p> <p>Level 4 Students should be able to describe a sequence that exhibits the similarity between two shapes and understand that the angle measures are unchanged.</p>

Grade 8 Mathematics Item Specification C1 TG

Evidence Required:	<ol style="list-style-type: none"> 1. The student verifies that rigid transformations preserve distance and angle measures. 2. The student describes sequences of rotations, reflections, translations, and dilations that can verify whether two-dimensional figures are similar or congruent to each other. 3. The student constructs a new figure that is the result of dilating, rotating, reflecting, or translating the original figure. 4. The student describes the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.
Allowable Response Types:	Matching Tables; Equation/Numeric; Hot Spot; Graphing
Allowable Stimulus Materials:	two-dimensional geometric figures, the coordinate plane, two-dimensional geometric figures in the coordinate plane
Construct-Relevant Vocabulary:	angle, transformation, translation, translate, rotation, rotate, reflection, reflect, dilation, dilate, line segment, similar, congruent, parallel, transversal, exterior angle, interior angle, angle-angle criterion, scale factor, vertical angles, adjacent angle, supplementary angles, complementary angles
Allowable Tools:	Calculator
Target-Specific Attributes:	<p>Rotations are only multiples of 90 degrees about the origin.</p> <p>Reflections are only over the x- and y-axes.</p> <p>Dilations are only with the origin as the center.</p> <p>There are at most three transformations in a sequence of transformations.</p> <p>Transformations are limited to the coordinate plane.</p>
Non-Targeted Constructs:	<p>Function notation for transformations</p> <p>Function rules for transformations</p> <p>Two-column proofs</p>
Accessibility Guidance:	<p>Item writers should consider the following Language and Visual Element/Design guidelines¹ when developing items.</p> <p>Language Key Considerations:</p> <ul style="list-style-type: none"> • Use simple, clear, and easy-to-understand language needed to assess the construct or aid in the understanding of the context • Avoid sentences with multiple clauses • Use vocabulary that is at or below grade level • Avoid ambiguous or obscure words, idioms, jargon, unusual names and references <p>Visual Elements/Design Key Considerations:</p> <ul style="list-style-type: none"> • Include visual elements only if the graphic is needed to assess the construct or it aids in the understanding of the context • Use the simplest graphic possible with the greatest degree of contrast, and include clear, concise labels where necessary

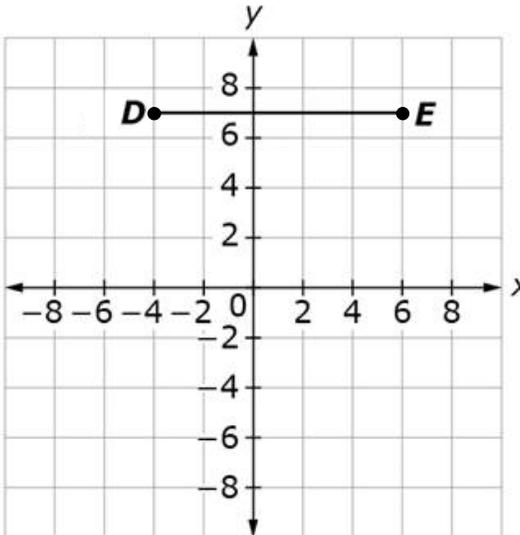
¹ For more information, refer to the General Accessibility Guidelines at:

<http://www.smarterbalanced.org/wordpress/wp-content/uploads/2012/05/TaskItemSpecifications/Guidelines/AccessibilityandAccommodations/GeneralAccessibilityGuidelines.pdf>

Grade 8 Mathematics Item Specification C1 TG

	<ul style="list-style-type: none"> • Avoid crowding of details and graphics <p>Items are selected for a student's test according to the blueprint, which selects items based on Claims and targets, not task models. As such, careful consideration is given to making sure fully accessible items are available to cover the content of every Claim and target, even if some item formats are not fully accessible using current technology.²</p>
Development Notes:	Much of the evidence for 8.G.A will be assessed in Claim 3.

² For more information about student accessibility resources and policies, refer to http://www.smarterbalanced.org/wordpress/wp-content/uploads/2014/08/SmarterBalanced_Guidelines.pdf

<p>Task Model 1</p> <p>Response Type: Equation/Numeric</p> <p>DOK Level 1</p> <p>8.G.A.1 Verify experimentally the properties of rotations, reflections, and translations: a. Lines are taken to lines, and line segments to line segments of the same length. b. Angles are taken to angles of the same measure. c. Parallel lines are taken to parallel lines.</p> <p>Evidence Required: 1. The student verifies that rigid transformations preserve distance and angle measures.</p> <p>Tools: Calculator</p>	<p>Prompt Features: The student is prompted to give a length or angle measure in a geometric figure after one rigid transformation.</p> <p>Stimulus Guidelines:</p> <ul style="list-style-type: none"> • The distance formula should not be needed to find the length of the line segment. • Angle measurements should be less than 180°. • Item difficulty can be adjusted via these example methods: <ul style="list-style-type: none"> ◦ Vary the type and number of transformations. ◦ Provide graphics that illustrate the transformation. ◦ Provide verbal descriptions of the transformation. ◦ The length of an original line segment is provided or student calculates the length from coordinates. ◦ The measure of an original angle is provided or student determines the angle measure. <p>TM1a Stimulus: The student is presented with an image or description of a geometric object and one rigid transformation.</p> <p>Example Stem 1: Line segment DE is translated left 3 units and down 2 units to form line segment $D'E'$.</p>  <p>Enter the distance, in units, between point D' and point E'.</p> <p>Example Stem 2: Line segment FG begins at $(-2, 4)$ and ends at $(-2, -3)$. The segment is translated left 3 units and up 2 units to form line segment $F'G'$.</p> <p>Enter the length, in units, of line segment $F'G'$.</p> <p>Rubric: (1 point) The student gives the correct measure (e.g., 10; 7).</p> <p>Response Type: Equation/Numeric</p>
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Task Model 1

Response Type:
Matching Tables

DOK Level 1

8.G.A.1

Verify experimentally the properties of rotations, reflections, and translations:

- a. Lines are taken to lines, and line segments to line segments of the same length.
- b. Angles are taken to angles of the same measure.
- c. Parallel lines are taken to parallel lines.

Evidence Required:

- 1. The student verifies that rigid transformations preserve distance and angle measures.

Tools: Calculator

Accessibility Note:

Presenting the coordinate plane without a grid and without tick marks, as suggested in the Stimulus Guidelines for varying difficulty, is not an accessibility issue for this particular task model given the purpose of the item.

Prompt Features: The student is prompted to verify that rigid transformations result in congruent figures.

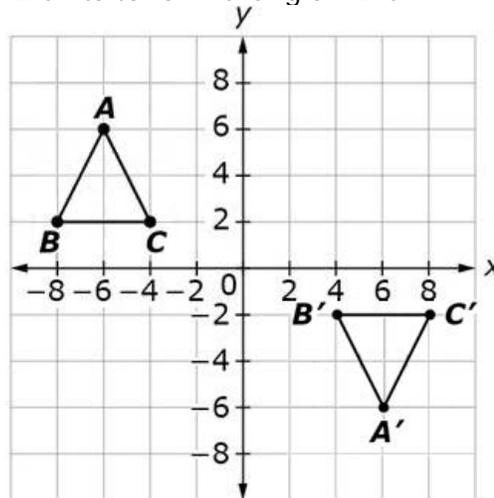
Stimulus Guidelines:

- The distance formula should not be needed to find the length of the line segment.
- Angle measurements should be less than 180° .
- Item difficulty can be adjusted via these example methods:
 - Vary the type and number of transformations
 - Vary the figure used
 - Having three options: True, False, Cannot be determined
 - Presenting the coordinate plane without grid and without tick marks

TM1b

Stimulus: The student is presented with two congruent figures on a coordinate plane and a description of a transformation.

Example Stem: Triangle ABC is reflected across the x -axis and then translated right 12 units to form triangle $A'B'C'$.



Select True or False for each statement.

Statement	True	False
Angle B has the same measure as angle B' .		
Side AC is longer than side $A'C'$.		
Side BC is the same length as side $B'C'$.		

Rubric: (1 point) The student correctly identifies all statements as True or False (e.g., T, F, T).

Response Type: Matching Tables

Task Model 1

Response Type:
Matching Tables

DOK Level 1

8.G.A.1

Verify experimentally the properties of rotations, reflections, and translations:

- a. Lines are taken to lines, and line segments to line segments of the same length.
- b. Angles are taken to angles of the same measure.
- c. Parallel lines are taken to parallel lines.

Evidence Required:

- 1. The student verifies that rigid transformations preserve distance and angle measures.

Tools: Calculator

Prompt Features: The student is prompted to match corresponding parts of figures after a rotation, reflection, and/or translation has been applied to a figure.

Stimulus Guidelines: Item difficulty can be adjusted via these example methods:

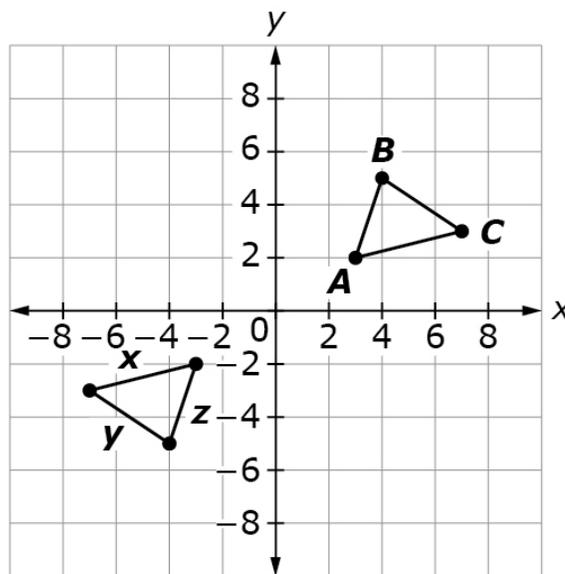
- Varying the type and number of transformations
- Varying the figure
- Providing graphics that illustrate the transformation
- Providing verbal descriptions of the transformation

TM1c

Stimulus: The student is presented with two congruent figures on a coordinate plane and a description of a transformation.

Example Stem: Triangle ABC was created by joining points $A(3, 2)$, $B(4, 5)$, and $C(7, 3)$ with line segments.

Triangle ABC is reflected over the x -axis and then reflected over the y -axis to form a triangle with side lengths x , y , and z .



Click in the table to show which side lengths are equal.

	x	y	z
AB			
AC			
BC			

Rubric: (1 point) The student correctly matches the sides of both triangles (e.g., $AB = z$, $AC = x$, $BC = y$).

Response Type: Matching Tables

Task Model 2

Response Type:
Matching Tables

DOK Level 2

8.G.A.2

Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.

8.G.3

Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

Evidence Required:

2. The student describes sequences of rotations, reflections, translations, and dilations that can verify whether two-dimensional figures are similar or congruent to each other.

Tools: Calculator

Prompt Features: The student is prompted to verify that two figures are similar or congruent by describing a sequence of rotations, reflections, translations, and dilations that exhibit the similarity or congruence between two given figures.

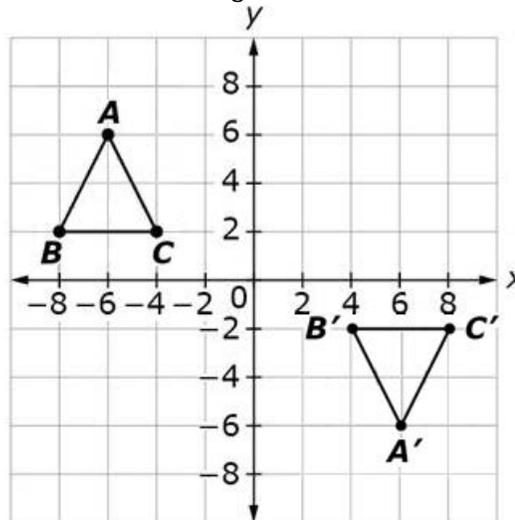
Stimulus Guidelines:

- A figure will contain no more than eight vertices.
- Item difficulty can be adjusted via these example methods:
 - Varying the type and number of transformations
 - Inclusion of dilations.

TM2

Stimulus: Transformations will include rotation, reflection, dilation, and/or translation.

Example Stem: Consider this figure.



Consider the statements in the table shown. Select True or False for each statement about the sequences of transformations that can verify that triangle ABC is congruent to triangle $A'B'C'$.

Statement	True	False
Triangle ABC is translated 12 units to the right, followed by a reflection across the x -axis.		
Triangle ABC is a reflected across the y -axis, followed by a translation 12 units down.		
Triangle ABC is reflected across the x -axis, followed by a translation 12 units to the right.		

Rubric: (1 point) The student selects True or False for the correct sequence of transformations for the figure (e.g., T, F, T).

Response Type: Matching Tables

<p>Task Model 3</p> <p>Response Type: Graphing</p> <p>DOK Level 2</p> <p>8.G.A.3 Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.</p> <p>Evidence Required: 3. The student constructs a new figure after the original figure is dilated, rotated, reflected, or translated.</p> <p>Tools: Calculator</p> <p>Accessibility Note: Graphing items are not currently able to be Brailled. Minimize the number of items developed to this TM.</p>	<p>Prompt Features: The student draws the image of a figure after a single rotation, reflection, translation, or dilation.</p> <p>Stimulus Guidelines:</p> <ul style="list-style-type: none"> • A figure will contain no more than eight vertices. • Item difficulty can be adjusted via these example methods: <ul style="list-style-type: none"> ○ Varying the type and number of transformations ○ Inclusion of dilations ○ Number of vertices <p>TM3</p> <p>Stimulus: The student is presented with a figure on a coordinate plane and a verbal description of a single rotation, reflection, translation, or dilation.</p> <p>Example Stem: The figure on the coordinate plane is reflected across the y-axis.</p> <div data-bbox="678 772 1250 1344" data-label="Figure"> <p>The figure shows a coordinate plane with a grid. The x-axis and y-axis both range from -8 to 8, with major grid lines every 2 units. A triangle is plotted in the second quadrant. The vertices of the triangle are labeled with their coordinates: (-8, 3), (-4, 3), and (-6, 7). The base of the triangle is horizontal, connecting (-8, 3) and (-4, 3). The third vertex is at (-6, 7).</p> </div> <p>Use the Connect Line tool to draw the resulting image of the figure.</p> <p>Interaction: The student uses the Connect Line tool to draw a figure on a grid. The Add Point and Delete button should also be available. The grid should have snap functions at every intersection of grid lines.</p> <p>Rubric: (1 point) The student draws the triangle in the correct location.</p> <p>Response Type: Graphing</p>
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Task Model 4

Response Type:
Hot Spot

DOK Level 2

8.G.A.3
Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

Evidence Required:
4. The student describes the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

Tools: Calculator

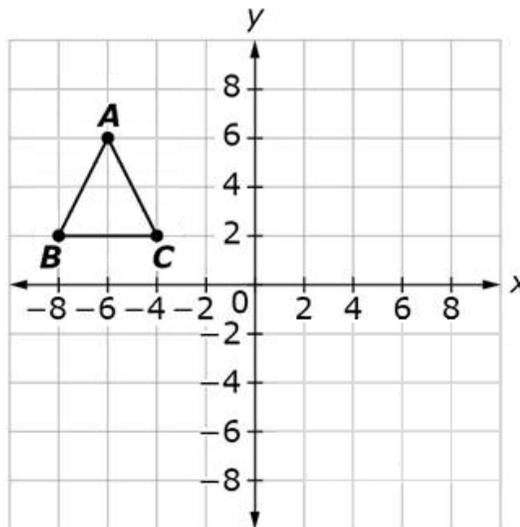
Accessibility Note:
Hot spot items are not currently able to be Brailled. Minimize the number of items developed to this TM.

Prompt Features: The student is prompted to give the coordinates of the image of a given point or set of points (or the quadrant that one or more coordinates is located in) after a sequence of transformations.

- Stimulus Guidelines:**
- Transformations can include rotation, reflection, translation, and dilation.
 - A figure should contain no more than eight vertices.
 - Item will ask for the coordinates of a point in the image of the figure after the transformation is applied, up to a maximum of three points.
 - Item difficulty can be adjusted via these example methods:
 - Varying the type and number of transformations
 - Inclusion of dilations
 - Number of vertices on figure
 - Number of coordinates that must be supplied by student.

TM4
Stimulus: The student is presented with a figure on a coordinate plane, along with a description of a sequence of transformations.

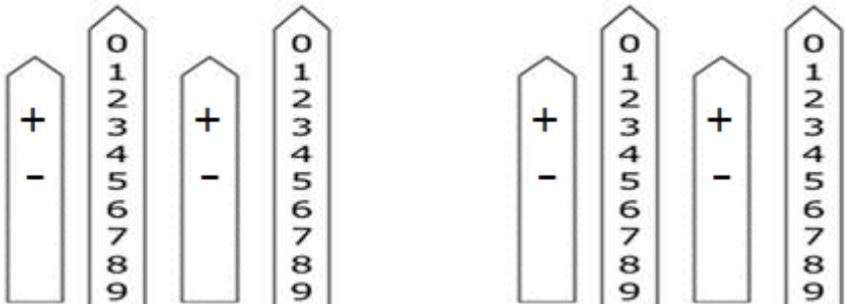
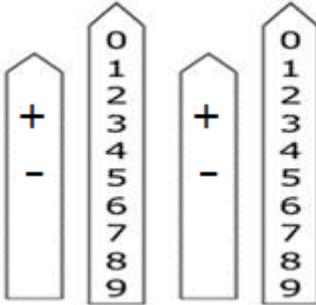
Example Stem: Triangle ABC is reflected across the x -axis, and dilated by a scale factor of 2, with the origin as the center of the dilation.



Click the numbers to give the coordinates of vertices $A'B'C'$.

Interaction: The student will click on numbers and positive/negative signs to give coordinates.

Grade 8 Mathematics Item Specification C1 TG

<p>Task Model 4</p> <p>Response Type: Hot Spot</p> <p>DOK Level 2</p> <p>8.G.A.3 Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.</p> <p>Evidence Required: 4. The student describes the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.</p> <p>Tools: Calculator</p> <p>Accessibility Note: Hot spot items are not currently able to be Brailled. Minimize the number of items developed to this TM.</p>	<p>$A' = (\quad , \quad)$ $B' = (\quad , \quad)$</p>  <p>$C' = (\quad , \quad)$</p>  <p>Rubric: (1 point) The student identifies all 3 points correctly. (e.g., $A' = (-12, -12)$, $B' = (-16, -4)$, $C' = (-8, -4)$).</p> <p>Response Type: Hot Spot</p>
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