

Grade 8 Mathematics Item Specification C1 TE

<p><b>Claim 1:</b> Concepts and Procedures Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.</p>	
<p>Content Domain: <b>Functions</b></p>	
<p><b>Target E [m]:</b> Define, evaluate, and compare functions. (DOK Levels 1, 2)</p> <p>Tasks associated with this target ask students to relate different representations of functions (equations, graphs, tables, or verbal descriptions). Some tasks for this target will ask students to produce or identify input and output pairs for a given function. Other tasks will ask students to compare properties of linear functions (e.g., rate of change or initial value).</p> <p>Other tasks should ask students to classify functions as linear or nonlinear when expressed in any of the representations listed above. Some of these may be connected to 8.SP Target J.</p>	
<p>Standards: 8.F.A, 8.F.A.1, 8.F.A.2, 8.F.A.3</p>	<p><b>8.F.A Define, evaluate, and compare functions.</b></p> <p><b>8.F.A.1</b> Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.</p> <p><b>8.F.A.2</b> Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). <i>For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.</i></p> <p><b>8.F.A.3</b> Interpret the equation <math>y = mx + b</math> as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. <i>For example, the function <math>A = s^2</math> giving the area of a square as a function of its side length is not linear because its graph contains the points <math>(1, 1)</math>, <math>(2, 4)</math> and <math>(3, 9)</math>, which are not on a straight line.</i></p>
<p>Related Below-Grade and Above-Grade Standards for Purposes of Planning for Vertical Scaling:</p> <p>7.RP.A, 7.RP.A.2c, 7.RP.A.2d, 7.EE.B, 7.EE.B.4, 7.EE.B.4a</p> <p>F-IF.A, F-IF.A.1, F-IF.A.2, F-IF.B, F-IF.B.4, F-IF.B.6, F-IF.C, F-IF.C.7, F-IF.C.7a, F-IF.C.7b, F-IF.C.7c, F-IF.C.8, F-IF.C.9</p>	<p><b>Related Grade 7 Standards</b></p> <p><b>7.RP.A Analyze proportional relationships and use them to solve real-world and mathematical problems.</b></p> <p><b>7.RP.A.2c</b> Represent proportional relationships by equations. <i>For example, if total cost <math>t</math> is proportional to the number <math>n</math> of items purchased at a constant price <math>p</math>, the relationship between the total cost and the number of items can be expressed as <math>t = pn</math>.</i></p> <p><b>7.RP.A.2d</b> Explain what a point <math>(x,y)</math> on the graph of a proportional relationship means in terms of the situation, with special attention to the points <math>(0, 0)</math> and <math>(1, r)</math> where <math>r</math> is the unit rate.</p> <p><b>7.EE.B Solve real-life and mathematical problems using numerical and algebraic expressions and equations.</b></p> <p><b>7.EE.B.4</b> Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.</p> <p><b>7.EE.B.4a</b> Solve word problems leading to equations of the form <math>px + q = r</math> and <math>p(x + q) = r</math>, where <math>p, q,</math> and <math>r</math> are specific rational numbers. Solve equations of these forms fluently. Compare an</p>

	<p>algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. <i>For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?</i></p> <p><b>Related High School Standards</b></p> <p><b>F-IF.A Understand the concept of a function and use function notation.</b></p> <p><b>F-IF.A.1</b> Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If <math>f</math> is a function and <math>x</math> is an element of its domain, then <math>f(x)</math> denotes the output of <math>f</math> corresponding to the input <math>x</math>. The graph of <math>f</math> is the graph of the equation <math>y = f(x)</math>.</p> <p><b>F-IF.A.2</b> Use function notations, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.</p> <p><b>F-IF.B Interpret functions that arise in applications in terms of the context.</b></p> <p><b>F-IF.B.4</b> For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. <i>Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.</i></p> <p><b>F-IF.B.6</b> Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.</p> <p><b>F-IF.C Analyze functions using different representations.</b></p> <p><b>F-IF.C.7</b> Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.</p> <p><b>F-IF.C.7a</b> Graph linear and quadratic functions and show intercepts, maxima, and minima.</p> <p><b>F-IF.C.7b</b> Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.</p> <p><b>F-IF.C.7c</b> Graph polynomial functions, identifying zeroes when suitable factorizations are available, and showing end behavior.</p> <p><b>F-IF.C.8</b> Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.</p> <p><b>F-IF.C.9</b> Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). <i>For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.</i></p>
DOK Levels:	1, 2
<b>Achievement Level Descriptors:</b>	
<b>RANGE Achievement Level Descriptor</b>	<b>Level 1</b> Students should be able to identify whether or not a relationship that is represented graphically, in a table, or algebraically, is a function. They should be able to compare the

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<p><b>(Range ALD)</b> Target E: Define, evaluate, and compare functions.</p>	<p>properties of two linear functions represented in the same way (graphically or in a table).</p> <p><b>Level 2</b> Students should be able to produce input and output pairs for a given function and identify whether an input/output pair satisfies a function. They should be able to compare properties of two functions represented in the same way (algebraic, graphic, tabular, or verbal). They should be able to classify functions as linear or nonlinear on the basis of their graph.</p> <p><b>Level 3</b> Students should be able to classify functions as linear or nonlinear in different forms (e.g., graphical, algebraic, verbal description, and/or tabular) and should know linear equations of the form <math>y = mx + b</math> are functions. They should also be able to define a function as a rule that assigns to each input exactly one output. They should be able to compare properties of two functions represented in different ways (algebraic, graphic, tabular, or verbal).</p> <p><b>Level 4</b> Students should be able to give examples of functions that are not linear and be able to compare properties of two nonlinear functions represented in different ways (algebraic, graphic, tabular, or verbal).</p>
<p>Evidence Required:</p>	<ol style="list-style-type: none"> <li>1. The student recognizes that a function is a rule that assigns to each input exactly one output.</li> <li>2. The student identifies or produces input and output pairs for given functions.</li> <li>3. The student recognizes the same function written in different functional forms (algebraic, graphic, tabular, or verbal).</li> <li>4. The student compares properties of two functions, each represented in a different way (algebraic, graphic, tabular, or verbal).</li> <li>5. [Retired this Evidence Required statement]</li> <li>6. The student recognizes and gives examples of functions that are not linear.</li> </ol>
<p>Allowable Response Types:</p>	<p>Multiple Choice, multiple correct response; Multiple Choice, single correct response; Equation/Numeric; Matching Tables</p>
<p>Allowable Stimulus Materials:</p>	<p>equations that define functions, graphical representations of functions, functions represented by tables, written descriptions of functions, sets of ordered pairs</p>
<p>Construct-Relevant Vocabulary:</p>	<p>function, relation, linear, nonlinear, ordered pairs, coordinate grid, rate of change, <math>y</math>-intercept, <math>x</math>-intercept, slope</p>
<p>Allowable Tools:</p>	<p>Calculator</p>
<p>Target-Specific Attributes:</p>	
<p>Non-Targeted Constructs:</p>	

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<p>Accessibility Guidance:</p>	<p>Item writers should consider the following Language and Visual Element/Design guidelines<sup>1</sup> when developing items.</p> <p>Language Key Considerations:</p> <ul style="list-style-type: none"> <li>• Use simple, clear, and easy-to-understand language needed to assess the construct or aid in the understanding of the context</li> <li>• Avoid sentences with multiple clauses</li> <li>• Use vocabulary that is at or below grade level</li> <li>• Avoid ambiguous or obscure words, idioms, jargon, unusual names and references</li> </ul> <p>Visual Elements/Design Key Considerations:</p> <ul style="list-style-type: none"> <li>• Include visual elements only if the graphic is needed to assess the construct or it aids in the understanding of the context</li> <li>• Use the simplest graphic possible with the greatest degree of contrast, and include clear, concise labels where necessary</li> <li>• Avoid crowding of details and graphics</li> </ul> <p>Items are selected for a student’s test according to the blueprint, which selects items based on Claims and targets, not task models. As such, careful consideration is given to making sure fully accessible items are available to cover the content of every Claim and target, even if some item formats are not fully accessible using current technology.<sup>2</sup></p>
<p>Development Notes:</p>	<p>Function notation is not required in grade 8. Items and tasks developed for the grade 8 Function domain should not use formal function notation [i.e., <math>f(x)</math>].<sup>3</sup></p>

<sup>1</sup> For more information, refer to the General Accessibility Guidelines at:

<http://www.smarterbalanced.org/wordpress/wp-content/uploads/2012/05/TaskItemSpecifications/Guidelines/AccessibilityandAccommodations/GeneralAccessibilityGuidelines.pdf>

<sup>2</sup> For more information about student accessibility resources and policies, refer to

[http://www.smarterbalanced.org/wordpress/wp-content/uploads/2014/08/SmarterBalanced\\_Guidelines.pdf](http://www.smarterbalanced.org/wordpress/wp-content/uploads/2014/08/SmarterBalanced_Guidelines.pdf)

<sup>3</sup> Items that use function notation may appear on the tests of high performing students as the adaptive algorithm allows for higher grade-level items to be selected.

**Task Model 1**

**Response Type:**  
Multiple Choice,  
single correct  
response

**DOK Level 1**

**8.F.A.1**

Understand that a function is a rule that assigns to each input exactly one output.

The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.

**Evidence Required:**

1. The student recognizes that a function is a rule that assigns to each input exactly one output.

**Tools:** Calculator

**Prompt Features:** The student identifies a function as a rule that assigns each input value to exactly one output value.

**Stimulus Guidelines:**

- Linear equations should be in either  $y = mx + b$  where  $b \neq 0$ , or standard form.
- Equations may include exponents or absolute value.
- Tables and graphs must be labeled.
- In general, items should make clear which is the input variable and which is the output variable. One exception: it is permissible to talk about “the function  $y = 2x + 3$ .” But it should not be assumed, for example, that values in a table have predefined roles as input or output.

**TM1a**

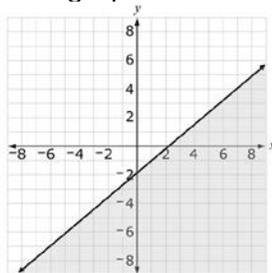
**Stimulus:** The student is presented with relations that may be functions and are represented as tables, graphs, and equations.

**Example Stem:** Which relation defines  $y$  as a function of  $x$ ?

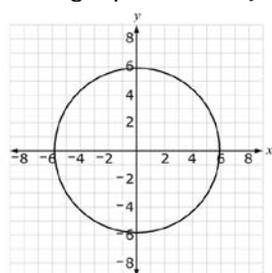
- A. The proportional relationship  $y = 2.4x$ .
- B. The table showing the age in years,  $x$ , and weight in pounds,  $y$ , of five dogs.

$x$	$y$
3	30
4	38
4	21
5	9
6	42

- C. The graph of an inequality as shown by the shaded region.



- D. The graph of  $x^2 + y^2 = 36$  as shown.



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<p><b>Task Model 1</b></p> <p><b>Response Type:</b> <b>Multiple Choice, multiple correct response</b></p> <p><b>DOK Level 1</b></p> <p><b>8.F.A.1</b> Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.</p> <p><b>Evidence Required:</b> 1. The student recognizes that a function is a rule that assigns to each input exactly one output.</p> <p><b>Tools:</b> Calculator</p> <p><b>Version 3 Update:</b> Renamed TM1 as TM1a and added new TM1b</p>	<p><b>Rubric:</b> (1 point) The student selects the relation that is a function (e.g., A).</p> <p><b>Response Type:</b> Multiple Choice, single correct response</p> <p><b>Prompt Features:</b> The student identifies a function as a rule that assigns each input value to exactly one output value.</p> <p><b>Stimulus Guidelines:</b></p> <ul style="list-style-type: none"> <li>• Linear equations should be in either <math>y = mx + b</math> where <math>b \neq 0</math>, or standard form.</li> <li>• Equations may include exponents or absolute value.</li> <li>• Tables and graphs must be labeled.</li> <li>• In general, items should make clear which is the input variable and which is the output variable. One exception: it is permissible to talk about “the function <math>y = 2x + 3</math>.” But it should not be assumed, for example, that values in a table have predefined roles as input or output.</li> </ul> <p><b>TM1b</b> <b>Stimulus:</b> The student is presented with equations in two variables.</p> <p><b>Example Stem:</b> Which equation defines <math>p</math> as a function of <math>t</math>?</p> <p>A. <math>p = 3t + 2</math> B. <math>t = 3p + 2</math> C. <math>p = 0t + 2</math> D. <math>t = 0p + 2</math></p> <p><b>Rubric:</b> Student selects correct values (e.g., A, B, C).</p> <p><b>Response Type:</b> Multiple Choice, multiple correct response</p>
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<p><b>Task Model 2</b></p> <p><b>Response Type:</b> <b>Multiple Choice, multiple correct response</b></p> <p><b>DOK Level 1</b></p> <p><b>8.F.A.1</b> Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.</p> <p><b>Evidence Required:</b> 2. The student identifies or produces input and output pairs for given functions.</p> <p><b>Tools:</b> Calculator</p>	<p><b>Prompt Features:</b> The student identifies or produces input and output pairs for given functions.</p> <p><b>Stimulus Guidelines:</b></p> <ul style="list-style-type: none"> <li>• Tables and graphs must be labeled.</li> <li>• Linear equations should be in either <math>y = mx + b</math> where <math>b \neq 0</math>, or standard form.</li> <li>• Context should be familiar to 13 to 15yearolds.</li> <li>• Item difficulty can be adjusted via these example methods:             <ul style="list-style-type: none"> <li>○ Elements in the ordered pair include integers or fractions.</li> <li>○ Equations may include exponents or absolute value.</li> </ul> </li> </ul> <p><b>TM2</b> <b>Stimulus:</b> The student is presented with an equation or table that represents a function.</p> <p><b>Example Stem 1:</b> Select <b>all</b> ordered pairs that correspond to input-output pairs for the function <math>y = -6x + 7</math>.</p> <p>A. (1, 1) B. (-1, 1) C. (-6, 7) D. (3, -11)</p> <p><b>Example Stem 2:</b> A swimming pool had 30 gallons of water in it. Then water was added to the pool at a rate of 5 gallons per second.</p> <p>The function <math>y = 5t + 30</math> describes the relationship between the number of gallons, <math>y</math>, and the number of seconds water was added, <math>t</math>.</p> <p>Select <b>all</b> of the ordered pairs that correspond to input-output pairs for the function.</p> <p>A. (45, 3) B. (3, 45) C. (0, 30) D. (30, 0)</p> <p><b>Answer Choices:</b> The answer choices will include at least two correct input-output pairs. Incorrect options will include input-output pairs resulting from switching the variables and integer signs, and using the slope and <math>y</math>-intercept to form an input-output pair.</p> <p><b>Rubric:</b> (1 point) The student selects all correct input-output pairs (e.g., A and D; B and C).</p> <p><b>Response Type:</b> Multiple Choice, multiple correct response</p>
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<p><b>Task Model 3</b></p> <p><b>Response Type:</b> <b>Multiple Choice, single correct response</b></p> <p><b>DOK Level 2</b></p> <p><b>8.F.A.2</b> Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). <i>For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.</i></p> <p><b>Evidence Required:</b> 3. The student recognizes the same function written in different functional forms (algebraic, graphic, tabular, or verbal).</p> <p><b>Tools:</b> Calculator</p> <p><b>Version 3 Update:</b> Revised TM3 Example Stem 1.</p>	<p><b>Prompt Features:</b> The student identifies the same function represented in different ways.</p> <p><b>Stimulus Guidelines:</b></p> <ul style="list-style-type: none"> <li>• Context should be familiar for 13 to 15 year olds.</li> <li>• Tables and graphs must be labeled.</li> <li>• Item difficulty can be adjusted via this example method:             <ul style="list-style-type: none"> <li>○ Representations in the answer choices are all equations, all tabular, all graphs, or verbal statements or a combination.</li> </ul> </li> </ul> <p><b>TM3</b> <b>Stimulus:</b> The student is presented with a function represented in algebraic, graphic, or tabular form.</p> <p><b>Example Stem 1:</b> Consider the function represented by this table of values.</p> <table border="1" style="margin: 10px auto;"> <thead> <tr> <th><math>x</math></th> <th><math>y</math></th> </tr> </thead> <tbody> <tr> <td>-4</td> <td>-10</td> </tr> <tr> <td>-3</td> <td>-7</td> </tr> <tr> <td>-2</td> <td>-4</td> </tr> <tr> <td>-1</td> <td>-1</td> </tr> <tr> <td>0</td> <td>2</td> </tr> </tbody> </table> <p>Which function could have produced the values in the table?</p> <p>A. <math>y = -x - 14</math>            B. <math>y = -3x + 2</math>            C. <math>y = 3x - 22</math>            D. <math>y = 3x + 2</math></p> <p><b>Rubric:</b> (1 point) The student selects the correct answer choice (e.g., D).</p> <p><b>Response Type:</b> Multiple Choice, single correct response</p> <p><b>Example Stem 2:</b> A swimming pool has 30 gallons of water in it. Water is added to the pool at a rate of 5 gallons per second.</p> <p>Which equation models the relationship between <math>W</math>, the number of gallons of water, and <math>t</math>, the number of seconds water is being added to the swimming pool?</p> <p>A. <math>W = 30t + 5</math>            B. <math>W = 5t + 30</math>            C. <math>W = t + 35</math>            D. <math>W = 35t</math></p> <p><b>Rubric:</b> (1 point) The student selects the correct answer (e.g., B).</p> <p><b>Response Type:</b> Multiple Choice, single correct response</p>	$x$	$y$	-4	-10	-3	-7	-2	-4	-1	-1	0	2
$x$	$y$												
-4	-10												
-3	-7												
-2	-4												
-1	-1												
0	2												

**Task Model 4**

**Response Type:**  
Multiple choice,  
single correct  
response

**DOK Level 2**

**8.F.A.2**

Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*

**Evidence Required:**

4. The student compares properties of two functions, each represented in a different way (algebraic, graphic, tabular, or verbal).

**Tools:** Calculator

**Version 3 Update:** Changed the response format for TM4 from Equation/Numeric to Multiple choice, single correct response and removed example stem 2.

**Prompt Features:** The student identifies one or more correct comparisons between two functions represented in different ways.

**Stimulus Guidelines:**

- Comparisons can involve specific values, rates of change, intercepts, starting points, etc.
- Rates of change must be positive unless a specific, real-world context is used in a problem.
- Context should be familiar to students 13 to 15 years old.
- Linear equations should be in either  $y = mx + b$  or standard form.
- Graphs may feature one quadrant or all four quadrants.
- Verbal descriptions are of linear functions or data.
- Item difficulty can be adjusted via these example methods:
  - One function is described verbally.
  - Function is given in algebraic form and compared to a function given in tabular, graph, or verbal form.
  - The property to compare is explicitly shown or described in the representation.
  - The property to compare is not explicitly shown or described in the representation.

**TM4**

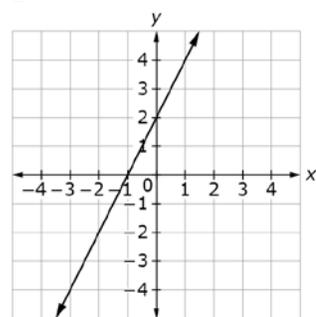
**Stimulus:** The student is presented with linear functions represented in different ways.

**Example Stem:** Each relation shown defines  $y$  as a function of  $x$ . Which function has the greatest rate of change?

- A.  $y = \frac{7}{3}x + 4$   
 B.  $y = 3x - 1$   
 C.

$x$	$y$
0	4
2	12
4	20
6	28

D.



**Rubric:** (1 point) The student identifies the correct function (e.g., C).

**Response Type:** Multiple choice, single correct response

**Task Model 6**

**Response Type:**  
Matching Tables

**DOK Level 2**

**8.F.A.3**

Interpret the equation  $y = mx + b$  as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function  $A = s^2$  giving the area of a square as a function of its side length is not linear because its graph contains the points (1, 1), (2, 4) and (3, 9), which are not on a straight line.*

**Evidence Required:**

6. The student recognizes and gives examples of functions that are not linear.

**Tools:** Calculator

**Version 3 Update:**  
Retired TM5

**Prompt Features:** The student recognizes representations of nonlinear functions.

**Stimulus Guidelines:**

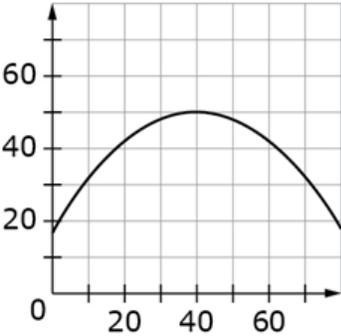
- Tables and graphs should be labeled.
- Tables should include 3–5 sets of values.
- Linear equations can be either  $y = mx + b$ , where  $b \neq 0$ , or  $y = ax + by + c = 0$ , where  $a > 0$  and  $c \neq 0$ .
- Nonlinear functions can include the forms  $y = x^2$ ,  $y = |x|$ , and  $y = \sqrt{x}$  where  $x > 0$ .
- Item difficulty can be adjusted via these example methods:
  - Functions may be represented as tables or equations rather than graphs.

**TM6**

**Stimulus:** The student is presented with linear and nonlinear functions represented in different ways.

**Example Stem:** Several functions are represented in the table.

Determine whether each function could be linear.

Function	Could be linear	Cannot be linear												
$y = \frac{3}{4}x + 2$														
														
<table border="1"> <thead> <tr> <th>x</th> <th>y</th> </tr> </thead> <tbody> <tr> <td>-2</td> <td>5</td> </tr> <tr> <td>-1</td> <td>9</td> </tr> <tr> <td>0</td> <td>13</td> </tr> <tr> <td>1</td> <td>17</td> </tr> <tr> <td>2</td> <td>21</td> </tr> </tbody> </table>	x	y	-2	5	-1	9	0	13	1	17	2	21		
x	y													
-2	5													
-1	9													
0	13													
1	17													
2	21													

**Rubric:** (1 point) The student selects the correct box to identify whether the functions are linear or nonlinear (e.g., L, N, L).

**Response Type:** Matching Tables