3) Apply properties of operations to $y + y + y$ to produce the equivalent expression $3y$.

4. Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them).

   Example: *The expressions $y + y + y$ and $3y$ are equivalent because they name the same number regardless of the numeric value of y.*

☐ **6.EE.B. Reason about and solve one-variable equations and inequalities.**

5. Understand solving an equation or inequality as a process of answering a question: Which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.
6. Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or depending on the purpose at hand, any number in a specified set.
7. Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q, and x are all nonnegative rational numbers.
8. Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem.
   a. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions.
   b. Represent solutions of such inequalities on number line diagrams.

☐ **6.EE.C. Represent and analyze quantitative relationships between two variables.**

9. Use variables to represent two quantities in a real-world problem that change in relationship to one another; write equations to represent the relationship between the two quantities. Analyze the relationship using graphs and tables and relate these to the equations. Include an understanding of independent and dependent variables.

   Examples:

   1) In a problem involving mixing water (W) and orange concentrate (C) to make a consistent flavor of orange juice, list and graph ordered pairs of cups of water and orange concentrate, and write the equations (e.g., $C = \frac{1}{2} \cdot W$ or $W = 2 \cdot C$) to represent the relationship between water (W) and orange concentrate (C).

2) When examining the relationship between time and the growth of a plant, time tends to be thought of as the independent variable and the height of the plant tends to be thought of as the dependent variable.

Seventh Grade Standards: Expressions and Equations

☐ 7.EE.A. Use properties of operations to generate equivalent expressions.

1. Apply properties of operations to add, subtract, factor, and expand linear expressions with rational coefficients.

   ***Example**: $4x + 2 = 2(2x + 1)$ and $-3\left(x - \frac{5}{3}\right) = -3x + 5$*

2. Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related.

   ***Examples**:*

   1) $a + 0.05a = 1.05a$ means that "increase by 5%" is the same as "multiply by 1.05."

   2) A shirt at a clothing store is on sale for 20% off the regular price, p. The discount can be expressed as $0.2p$. The new price for the shirt can be expressed as $p - 0.2p$ or $0.8p$.

☐ 7.EE.B. Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

3. Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (integers, fractions, and decimals). Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.

   ***Examples**:*

   1) If a woman making $\$25$ per hour gets a 10% raise, she will make an additional $\frac{1}{10}$ of her salary an hour, or $\$2.50$, for a new salary of $\$27.50$.

   2) If you want to place a towel bar $9\frac{3}{4}$ inches long in the center of a door that is $27\frac{1}{2}$ inches wide, you will need to place the bar about nine inches from each edge; this estimate can be used as a check on the exact computation.

4. Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

    a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach.

***Example**: The perimeter of a rectangle is 54 cm. Its width is 6 cm. What is its length?*

    b. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem.

***Example**: As a salesperson, you are paid $\$50$ per week plus $\$3$ per sale. This week you want your pay to be at least $\$100$. Write an inequality for the number of sales you need to make, and describe the solutions.*

Eighth Grade Standards: Expressions and Equations

☐ **8.EE.A. Work with radicals and integer exponents.**

1. Know and apply the properties of integer exponents to generate equivalent numerical expressions.

***Example**: $3^2 \times 3^{-5} = 3^{-3} = \left(\frac{1}{3}\right)^3 = \frac{1}{27}$*

2. Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.
3. Use numbers expressed in the form of a single digit multiplied by an integer power of ten (scientific notation) to estimate very large or very small quantities, and express how many times as much one is than the other.

***Example**: Estimate the population of the United States as 3×10^8 and the population of the world as 7×10^9, and determine that the world population is more than 20 times larger.*

4. Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose

units of appropriate size for measurements of very large or very small quantities. Interpret scientific notation that has been generated by technology.

***Example**: Millimeters per year for seafloor spreading*

☐ **8.EE.B. Understand the connections between proportional relationships, lines, and linear equations.**

5. Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.

   ***Example**: Compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.*

6. Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane. Derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.

☐ **8.EE.C. Analyze and solve linear equations and pairs of simultaneous linear equations.**

7. Solve linear equations in one variable.

   a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$ (1 solution), $a = a$ (infinitely many solutions), or $a = b$ (no solution) results (where a and b are different numbers).

   ***Example**: $-3x - 2 = 7x + 2 - 10x$ has no solution because the equation simplifies to $-2 = 2$ which is false for any value of x.*

   b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

8. Analyze and solve pairs of simultaneous linear equations.

   a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs because points of intersection satisfy both equations simultaneously.

b. Solve systems of two linear equations in two variables algebraically (including but not limited to using substitution and elimination strategies), and estimate solutions by graphing the equations; solve simple cases by inspection.

***Example**: $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.*

c. Solve real-world and mathematical problems leading to two linear equations in two variables.

***Examples**:*

1) Given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.

2) Your family decided to rent a snowmobile at Island Park. Company A charges $\$125$ for the first hour plus $\$37.50$ for each additional hour. Company B charges a $\$50$ one-time rental fee plus $\$45$ per hour. Which company would cost less for you to rent for 3 hours? 5 hours? 8 hours?

Pages 128 through 135 in the Idaho Content Standards for Mathematics show how the Algebra Domain extends into Grades 9 through 12.

FUNCTIONS GRADE 8

Eighth Grade Standards: Functions

☐ **8.F.A. Define, evaluate, and compare functions.**

1. Understand that a function is a rule that assigns to each input exactly one output and that the graph of a function is the set of ordered pairs consisting of an input and the corresponding output.
2. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

   ***Example**: Given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*

3. Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear.

   ***Example**: The function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points $(1, 1)$, $(2, 4)$ and $(3, 9)$, which are not on a straight line.*

☐ **8.F.B. Use functions to model relationships between quantities.**

4. Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.
5. Describe qualitatively the functional relationship between two quantities by analyzing and sketching a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Pages 136 through 144 in the Idaho Content Standards for Mathematics show how the Functions Domain extends into Grades 9 through 12.

OPERATIONS: MASTERY OF BASIC CALCULATION GRADES K – 7

Note: These standards are listed in the Idaho Content Standards as Mastery Standards. Mastery standards describe those standards that ask students to be able to perform mathematical calculations accurately, efficiently, and flexibly. For standards related to knowing single-digit facts from memory, this typically involves generating a response within three to five seconds.

Kindergarten Standards: Mastery of Basic Calculation
K.OA.A.5 Fluently add and subtract within five, including zero.

First Grade Standards: Mastery of Basic Calculation
1.OA.C.6 Demonstrate fluency for addition and subtraction within ten, use strategies to add and subtract within 20.

Second Grade Standards: Mastery of Basic Calculation
2.OA.B.2 Demonstrate fluency for addition and subtraction within 20 using mental strategies. By the end of grade two, recall basic facts to add and subtract within 20 with automaticity. 2.NBT.B.5 Fluently add and subtract whole numbers within 100 using understanding of place value and properties of operations.

Third Grade Standards: Mastery of Basic Calculation
3.OA.C.7.b Demonstrate fluency for multiplication within 100. Know from memory all products of two single-digit numbers and related division facts. 3.NBT.A.2* Fluently add and subtract whole numbers within 1,000 using understanding of place value and properties of operations. **Designated as a mastery standard because students in third grade fluently add and subtract within 1,000 using methods based on place value, properties of operations, and/or the relationship between addition and subtraction. They focus on methods that generalize readily to larger numbers, emphasizing the relationship between addition and*

subtraction. These methods can be extended to 1,000,000 in fourth grade and fluency can be developed with larger numbers.

Fourth Grade Standards: Mastery of Basic Calculation
4.NBT.B.4 Fluently use the standard algorithm for multi-digit whole-number addition and subtraction.

Fifth Grade Standards: Mastery of Basic Calculation
5.NBT.B.5 Demonstrate fluency for multiplication of multi-digit whole numbers using the standard algorithm. Include two-digit × four-digit numbers and three-digit × three-digit numbers.

Sixth Grade Standards: Mastery of Basic Calculation
6.NS.B.2 Fluently divide multi-digit numbers using the standard algorithm. 6.NS.B.3 Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

Seventh Grade Standards: Mastery of Basic Calculation
7.NS.A.3 Solve real-world and mathematical problems involving the four operations with integers and other rational numbers.

OPERATIONS: ADDITION AND SUBTRACTION GRADES K – 7

Kindergarten Standards: Addition and Subtraction

☐ **K.OA.A. Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.**

1. Represent addition and subtraction of two whole numbers within ten. Use objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.
2. Solve addition and subtraction word problems within ten by using physical, visual, and symbolic representations.

   Clarification: *Students are not expected to independently read word problems.*
3. Decompose whole numbers from one to ten into pairs in more than one way by using physical, visual, or symbolic representations.

   Example*: Decomposing* 5 *may include* $5 = 2 + 3$ *and* $5 = 4 + 1$.
4. 4. For a given whole number from one to nine, find the number that makes ten when added to the number by using physical, visual, or symbolic representations
5. Fluently add and subtract within five, including zero.

   Clarification: *Fluency is reached when students are proficient, i.e., when they display accuracy, efficiency, and flexibility.*

First Grade Standards: Addition and Subtraction

☐ **1.OA.A. Represent and solve problems involving addition and subtraction.**

3. Solve addition and subtraction word problems within 20 involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, by using physical, visual, and symbolic representations.
4. Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20 by using physical, visual, and symbolic representations.

   Clarification: *Students are not expected to independently read word problems.*

☐ **1.OA.B. Understand and apply properties of operations and the relationship between addition and subtraction.**

5. Apply properties of operations to add.

***Examples**:*

1) If $8 + 3 = 11$ is known, then $3 + 8 = 11$ is also known. (Commutative property of addition.)

2) To add $2 + 6 + 4$, the second two numbers can be added to make a ten, so $2 + 6 + 4 = 2 + 10 = 12$. (Associative property of addition.)

***Clarification:** Students need not use formal terms for these properties.*

5. Restate a subtraction problem as a missing addend problem using the relationship between addition and subtraction.

***Example:** The equation $12 - 7 = ?$ can be restated as $7 + ? = 12$ to determine the difference is 5.*

☐ 1.OA.C. Add and subtract within 20.

7. Relate counting to addition and subtraction.

***Example:** When students count on 3 from 4, they should write this as $4 + 3 = 7$. When students count on for subtraction, 3 from 7, they should connect this to $7 - 3 = 4$. Students write "$7 - 3 = ?$" and think "I count on $3 + ? = 7$."*

8. Demonstrate fluency for addition and subtraction within ten, use strategies to add and subtract within 20.

***Clarification:** Fluency is reached when students are proficient, i.e., when they display accuracy, efficiency, and flexibility.*

Students may use mental strategies such as counting on, making ten, decomposing a number leading to a ten, using the relationship between addition and subtraction, and creating equivalent but easier or known sums.

☐ 1.OA.D. Work with addition and subtraction equations.

9. Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false.

***Example:** Which of the following equations are true and which are false? $6 = 6$, $7 = 8 - 1$, $5 + 2 = 2 + 5$, $4 + 1 = 5 + 2$*

9. Determine the unknown whole number in an addition or subtraction equation relating three whole numbers, with the unknown in any position.

Example: *Determine the unknown number that makes the equation true in each of the equations* $8+?=11,\ 5=?-3,\ 6+6=?$.

☐ **1.NBT.C. Use place value understanding and properties of operations to add and subtract.**

4. Add whole numbers within 100 by using physical, visual, and symbolic representations, with an emphasis on place value, properties of operations, and/or the relationship between addition and subtraction.
    a. Add a two-digit number and a one-digit number.
    b. Add a two-digit number and a multiple of ten.
    c. Understand that when adding two-digit numbers, combine like base-ten units such as tens and tens, ones and ones, and sometimes it is necessary to compose a ten.
5. Given a two-digit number, mentally find ten more or ten less than the number, without having to count; explain the reasoning used.
6. Subtract multiples of ten in the range 10 – 90 from multiples of ten in the range 10 – 90 by using physical, visual, and symbolic representations, with an emphasis on place value, properties of operations, and/or the relationships between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

Example*:* $70-40$ *can be thought of as* 7 *tens take away* 4 *tens, or can be rewritten as a missing addend problem:* $40+?=70$.

Second Grade Standards: Addition and Subtraction

☐ **2.OA.A. Represent and solve problems involving addition and subtraction.**

9. Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, by using physical, visual, and symbolic representations.

☐ **2.OA.B. Add and subtract within 20.**

2. Demonstrate fluency for addition and subtraction within 20 using mental strategies. By the end of grade two, recall basic facts to add and subtract within 20 with automaticity.

Clarification: *Fluency is reached when students are proficient, i.e., when they display accuracy, efficiency, and flexibility.*

Students may use mental strategies such as counting on, making ten; decomposing a number leading to a ten; using the relationship between addition and subtraction, and creating equivalent but easier or known sums.

☐ **2.NBT.A. Understand place value.**

1. Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones. Understand:
    a. 100 can be thought of as a bundle of ten tens—called a "hundred."
    b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, and 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).

***Example**: The number* 241 *can be expressed as* $2\ hundreds + 4\ tens + 1\ one$ *or as* $24\ tens + 1\ one$ *or as* $241\ ones$.

☐ **2.NBT.B. Use place value understanding and properties of operations to add and subtract.**

5. Fluently add and subtract whole numbers within 100 using understanding of place value and properties of operations.

***Clarification:** Fluency is reached when students are proficient, i.e., when they display accuracy, efficiency, and flexibility.*

6. Add up to four two-digit numbers using strategies based on place value and properties of operations.
7. Add and subtract whole numbers within 1,000, by using physical, visual, and symbolic representations, with an emphasis on place value, properties of operations, and/or the relationships between addition and subtraction.
    a. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones.
    b. Understand that sometimes it is necessary to compose or decompose tens or hundreds.

***Example:** Students may use equations to represent their strategies based on place value such as:* $324 + 515 = (300 + 500) + (20 + 10) + (4 + 5) = 839$.

8. Use mental strategies to add or subtract a number that is ten more, ten less, one hundred more, and one hundred less than a given three-digit number.

9. Explain why addition and subtraction strategies work, using place value and the properties of operations.

☐ **2.MD.B. Relate addition and subtraction to length.**

8. Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units.

Clarification: *Students may use drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem.*

9. Represent whole numbers as lengths from zero on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, ..., and represent whole-number sums and differences within 100 on a number line diagram.

Third Grade Standards: Addition and Subtraction

☐ **3.OA.D. Solve problems involving the four operations, and identify and explain patterns in arithmetic.**

10. Solve two-step word problems involving whole numbers using the four operations.
    a. Represent these problems using equations with a letter standing for the unknown quantity.
    b. Assess the reasonableness of answers using mental computation and estimation strategies, including rounding.
11. Identify arithmetic patterns (including patterns in the addition table or multiplication table) and explain them using properties of operations.

Example*: Arithmetic patterns are patterns that change by the same rate, such as adding the same number; the series* $2, 4, 6, 8, 10$ *is an arithmetic pattern that increases by* 2 *between each term.*

○ **3.NBT.A. Use place value understanding and properties of operations to perform multi-digit arithmetic**.

2. Round a whole number to the tens or hundreds place, using place value understanding or a visual representation.

2. Fluently add and subtract whole numbers within 1,000 using understanding of place value and properties of operations.

Clarification: *Fluency is reached when students are proficient, i.e., when they display accuracy, efficiency, and flexibility.*

Fourth Grade Standards: Addition and Subtraction

☐ **4.NBT.B. Use place value understanding and properties of operations to perform multi-digit arithmetic on whole numbers less than or equal to 1,000,000.**

4. Fluently use the standard algorithm for multi-digit whole-number addition and subtraction.

Example: *What is the difference between* 634 *and* 328 *using the standard algorithm?*

$$\begin{array}{r} 6\,{}^{2}\not{3}\,{}^{1}4 \\ -\,3\;2\;\;8 \\ \hline 3\;0\;\;6 \end{array}$$

Clarification*: Fluency is reached when students are proficient, i.e., when they display accuracy, efficiency, and flexibility.*

☐ **4.NF.B. Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.**

2. Understand a fraction $\frac{a}{b}$ with $a > 1$ as a sum of fractions $\frac{1}{b}$.

   a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.

   b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify the conclusions by using a visual fraction model and/or verbal reasoning.

Example: $\frac{3}{8} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$; $\frac{3}{8} = \frac{1}{8} + \frac{2}{8}$; $2\frac{1}{8} = 1 + 1 + \frac{1}{8} = \frac{8}{8} + \frac{8}{8} + \frac{1}{8}$

   c. Add and subtract mixed numbers with like denominators by replacing the mixed number with an equivalent fraction and/or by using properties of operations and the relationship between addition and subtraction.

   d. Solve word problems involving addition and subtraction of fractions, including mixed numbers, with the same denominator. Justify the conclusions using a visual fraction model and/or verbal reasoning.

Fifth Grade Standards: Addition and Subtraction

☐ 5.NBT.B. Perform operations with multi-digit whole numbers and with decimals to hundredths.

7. Add, subtract, multiply, and divide decimals to hundredths.
    a. Use concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction and between multiplication and division.
    b. Relate the strategy to a written method and explain the reasoning used.

Clarification for 5.NBT.B.6 and 5.NBT.B.7: *Students should be familiar with multiple strategies but should be able to select and use the strategy with which they most closely connect and understand, with the ultimate goal of supporting students to use more efficient strategies.*

☐ 5.NF.A. Use equivalent fractions as a strategy to add and subtract fractions.

1. Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions to produce an equivalent sum or difference of fractions with like denominators.

Example: $\frac{2}{3}+\frac{5}{4}=\frac{8}{12}+\frac{15}{12}=\frac{23}{12}$. *In general,* $\frac{a}{b}+\frac{c}{d}=\frac{ad+bc}{bd}$.

2. Solve word problems involving addition and subtraction of fractions referring to the same whole (the whole can be a set of objects), including cases of unlike denominators.
    a. Justify the conclusions by using visual fraction models and/or equations to represent the problem.
    b. Use benchmark fractions and number sense of fraction to estimate mentally and assess the reasonableness of answers.

Example: *Recognize an incorrect result* $\frac{2}{5}+\frac{1}{2}=\frac{3}{7}$ *by observing that* $\frac{3}{7}<\frac{1}{2}$.

Sixth Grade Standards: Addition and Subtraction

○ 6.NS.B. Compute fluently with multi-digit numbers and find common factors and multiples.

9. Fluently add, subtract, multiply, and divide multi-digit decimals using the

standard algorithm for each operation.

Example: *What is the difference of* 1.82 *and* 0.06 *using the standard algorithm?*

$$\begin{array}{r} 1.{}^{7}\cancel{8}{}^{1}2 \\ -\,0.\,0\;\;6 \\ \hline 1.\,7\;\;6 \end{array}$$

Seventh Grade Standards: Addition and Subtraction

☐ **7.NS.A. Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.**

1. Apply and extend previous understandings of addition and subtraction to add and subtract integers and other rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

    a. Describe situations in which opposite quantities combine to make zero.

Example*: If you open a new bank account with a deposit of* $\$30.52$ *and then withdraw* $\$30.52$*, you are left with a* $\$0$ *balance.*

    b. Understand $p + q$ as the number located a distance $|q|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite are additive inverses because they have a sum of 0 (e.g., $12.5 + (-12.5) = 0$). Interpret sums of rational numbers by describing real-world contexts.

    c. Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.

    d. Apply properties of operations as strategies to add and subtract rational numbers.

Example*:* $\frac{1}{4} - 5 + \frac{3}{4} + 7 = \left(\frac{1}{4} + \frac{3}{4}\right) + \big((-5) + 5\big) + 2$

OPERATIONS: MULTIPLICATION AND DIVISION GRADES 2 – 8

Second Grade Standards: Multiplication and Division

△ 2.OA.C. Work with equal groups of objects to gain foundations for multiplication.

3. Determine whether a group of objects (up to 20) has an odd or even number of members and write an equation to express an even number as a sum of two equal addends.

   Clarification: *Students may pair objects or count them by twos.*

4. Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.

   Example*: The total number of objects arranged in a* 2×5 *rectangular array can be found by adding* $2 + 2 + 2 + 2 + 2$.

Third Grade Standards: Multiplication and Division

☐ 3.OA.A. Represent and solve problems involving multiplication and division.

12. Interpret a product of whole numbers as a grouping of sets, e.g., 5×7 as five groups of seven objects each.
13. Interpret a quotient of whole numbers as equal sharing, e.g., $56 \div 8$ as the number in each share when 56 objects are split into 8 equal shares, or as the number of shares when 56 objects are split into equal shares of 8 objects each.
14. Use multiplication and division within 100 to solve word problems involving equal groups, arrays, and measurements by using visual and symbolic representations, with a symbol for an unknown number.
15. Determine the unknown whole number in a multiplication or division equation relating three whole numbers.

    Example*: Determine the unknown number that makes the equation true in each of the equations:* $8 \times ? = 48$, $5 = ? \div 3$, $6 \times 6 = ?$.

☐ 3.OA.B. Understand properties of multiplication and the relationship between multiplication and division.

16. Apply the properties of operations to multiply and divide.

Clarification: *Students need not use formal terms for these properties (identity, communicative, associative, distributive).*

17. Understand division as determining an unknown factor in a multiplication problem.

☐ **3.OA.C. Multiply and divide within 100.**

18. Demonstrate fluency for multiplication within 100.

    a. Demonstrate understanding of strategies that make use of the relationship between multiplication and division or properties of operations.

    b. Know from memory all products of two single-digit numbers and related division facts.

Clarification: *Fluency is reached when students are proficient, i.e., when they display accuracy, efficiency, and flexibility.*

☐ **3.OA.D. Solve problems involving the four operations, and identify and explain patterns in arithmetic.**

19. Solve two-step word problems involving whole numbers using the four operations.

    c. Represent these problems using equations with a letter standing for the unknown quantity.

    d. Assess the reasonableness of answers using mental computation and estimation strategies, including rounding.

20. Identify arithmetic patterns (including patterns in the addition table or multiplication table) and explain them using properties of operations.

Example*: Arithmetic patterns are patterns that change by the same rate, such as adding the same number; the series* $2, 4, 6, 8, 10$ *is an arithmetic pattern that increases by* 2 *between each term.*

○ **3.NBT.A. Use place value understanding and properties of operations to perform multi-digit arithmetic**.

3. Multiply one-digit whole numbers by multiples of ten in the range 10– 90 using understanding of place value and properties of operations.

☐ **3.MD.C. Geometric measurement: Understand concepts of area and relate area to multiplication and to addition.**

6. Relate area to the operations of multiplication and addition.

a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.

b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real-world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.

c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning.

Example: *Using the distributive property, the area of a shape that is* 6 *by* 7 *can be determined by finding the area of the* 6×5 *section and the* 6×2 *section and then adding the two products together.*

d. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real-world problems.

Example*: A pool is comprised of two non-overlapping rectangles in the shape of an "L." The area for a cover of a pool can be found by adding the areas of the two non-overlapping rectangles.*

○ 3.MD.D. Geometric measurement: Recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.

8. Solve real-world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.

Fourth Grade Standards: Multiplication and Division

☐ 4.OA.A. Use the four operations with whole numbers to solve problems.

4. Interpret a multiplication equation as a comparison, e.g., $35 = 5 \times 7$, as 35 is 5 times as many as 7. Represent verbal multiplicative comparisons as equations.
5. Multiply or divide to solve word problems involving multiplicative comparison.

Example*: If the cost of a red hat is three times more than a blue hat that costs* $\$5$*, then a red hat costs* $\$15$*.*

Clarification: *Students may use drawings and equations with a symbol for the unknown number to represent the problem.*

6. Distinguish between multiplicative comparison and additive comparison. Solve multi-step whole-number word problems using the four operations, including problems in which remainders must be interpreted.
    a. Represent these problems using equations with a letter standing for the unknown quantity.
    b. Assess the reasonableness of answers using mental computation and estimation strategies, including rounding.

△ 4.OA.B. Gain familiarity with factors and multiples.

6. Find all factor pairs for a whole number in the range 1–100.
    d. Recognize that a whole number is a multiple of each of its factors.
    e. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number.
    f. Determine whether a given whole number in the range 1–100 is prime or composite.

☐ 4.NBT.B. Use place value understanding and properties of operations to perform multi-digit arithmetic on whole numbers less than or equal to 1,000,000.

5. Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers.
    a. Use strategies based on place value and the properties of operations.
    b. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
6. Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors.
    a. Use strategies based on place value, the properties of operations, and/or the relationship between multiplication and division.
    b. Illustrate and explain the calculation by using rectangular arrays, area models, and/or equations.

Clarification for 4.NBT.B.5 and 4.NBT.B.6: *Students should be familiar with multiple strategies but should be able to select and use the strategy with which they most closely connect and understand, with the ultimate goal of supporting students to use more efficient strategies.*

☐ 4.NF.B. Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

6. Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.

    a. Understand a fraction $\frac{a}{b}$ as a multiple of $\frac{1}{b}$.

Example: *Use a visual fraction model to represent* $\frac{5}{4}$ *as the product* $5 \times \frac{1}{4}$*, recording the conclusion by the equation* $\frac{5}{4} = 5 \times \frac{1}{4}$.

    b. Understand a multiple of $\frac{a}{b}$ as a multiple of $\frac{1}{b}$, and use this understanding to multiply a fraction by a whole number.

Example: *Use a visual fraction model to express* $3 \times \frac{2}{5}$ *as* $6 \times \frac{1}{5}$*, recognizing this product as* $\frac{6}{5}$*. In general,* $n \times \frac{a}{b} = \frac{n \times a}{b}$.

    c. Solve word problems involving multiplication of a fraction by a whole number e.g., by using visual fraction models and/or equations to represent the problem.

Example: *If each person at a party will eat* $\frac{3}{8}$ *of a pound of roast beef, and there will be* 5 *people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?*

Fifth Grade Standards: Multiplication and Division

☐ 5.NBT.B. Perform operations with multi-digit whole numbers and with decimals to hundredths.

5. Demonstrate fluency for multiplication of multi-digit whole numbers using the standard algorithm. Include two-digit × four-digit numbers and three-digit × three-digit numbers.

Example: *What is the product of* 304 *and* 23 *using the standard algorithm?*

$$\begin{array}{r} {}^{3}{}^{1}0\,4 \\ \times\ 2\,3 \\ \hline 9\,1\,2 \\ +6\,0\,8\,0 \\ \hline 6\,9\,9\,2 \end{array}$$

Clarification*: Fluency is reached when students are proficient, i.e., when they display accuracy, efficiency, and flexibility.*

6. Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors.
    a. Use strategies based on place value, the properties of operations, and/or the relationship between multiplication and division.
    b. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
7. Add, subtract, multiply, and divide decimals to hundredths.
    a. Use concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction and between multiplication and division.
    b. Relate the strategy to a written method and explain the reasoning used.

☐ **5.NF.B. Apply and extend previous understandings of multiplication and division to multiply and divide fractions.**

3. Interpret a fraction as division of the numerator by the denominator $\left(\frac{a}{b} = a \div b\right)$. Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers by using visual fraction models and/or equations to represent the problem.

Example: *Interpret* $\frac{3}{4}$ *as the result of dividing* 3 *by* 4*, noting that* $\frac{3}{4}$ *multiplied by* 4 *equals* 3*, and that when* 3 *wholes are shared equally among* 4 *people, each person has a share of size* $\frac{3}{4}$*. If nine people want to share a* 50*-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?*

4. Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

    a. Interpret the product $\left(\frac{a}{b}\right) \times q$ as a parts of a partitions of q into b equal parts, and equivalently, as the result of the sequence of operations $a \times q \div b$.

Example: *Use a visual model and/or area model to show* $\left(\frac{2}{3}\right) \times 4 = \frac{8}{3}$*, and create a story context for this equation. Do the same with* $\left(\frac{2}{3}\right) \times \left(\frac{4}{5}\right) = \frac{8}{15}$*. In general,* $\left(\frac{a}{b}\right) \times \left(\frac{c}{d}\right) = \frac{ac}{bd}$*.*

    b. Find the area of a rectangle with fractional side lengths.

        i. Tile it with unit squares of the appropriate unit fraction side lengths.
        ii. Show that the area is the same by tiling as would be found by multiplying the side lengths.
        iii. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.

5. Interpret multiplication as scaling (resizing), by:

    a. Comparing the size of a fractional product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.

Example: *Without multiplying, tell which number is greater:* 225 *or* $\frac{3}{4} \times 225$*;* $\frac{11}{50}$ *or* $\frac{3}{2} \times \frac{11}{50}$*?*

    b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number, explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number, and relating the principle of fraction equivalence $\frac{a}{b} = \frac{n \times a}{n \times b}$ to the effect of multiplying $\frac{a}{b}$ by 1.

6. Solve real-world problems involving multiplication of fractions and mixed numbers by using visual fraction models and/or equations to represent the problem.

Example: *Evan bought six roses for his mother,* $\frac{2}{3}$ *of them were red. How many red roses were there?*

7. Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.

a. Represent division of a unit fraction by a nonzero whole number and compute such quotients using a visual fraction model. Use the relationship between multiplication and division to explain that $\frac{1}{b} \div c = \frac{1}{bc}$ because $\frac{1}{bc} \times c = \frac{1}{b}$.

Example: *Create a story context to explain* $\frac{1}{3} \div 4$*, and use a visual fraction model to show the quotient.*

b. Represent division of a whole number by a unit fraction, and compute such quotients using a visual fraction model. Use the relationship between multiplication and division to explain that $a \div \frac{1}{b} = ab$ because $ab \times \frac{1}{b} = a$.

Example: *Create a story context to explain* $4 \div \frac{1}{5}$*, and use a visual fraction model to show the quotient.*

c. Solve real-world problems involving division of unit fractions by nonzero whole numbers and division of whole numbers by unit fractions by using visual fraction models and/or equations to represent the problem.

Example: *How much chocolate will each person get if three people share* $\frac{1}{2}$ *lb of chocolate equally? How many* $\frac{1}{3}$ *cup servings are in two cups of raisins?*

Sixth Grade Standards: Application of Multiplication and Division

☐ **6.RP.A. Understand ratio and rate concepts and use ratio and rate reasoning to solve problems.**

1. Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.

Examples*:*

1) The ratio of wings to beaks in the bird house at the zoo was $2:1$*, because for every two wings there was one beak.*

2) For every vote candidate A received, candidate C received nearly three votes, meaning that candidate C received approximately three times the number of votes as candidate A, or candidate A received approximately $\frac{1}{3}$ *of the number of votes that candidate C received.*

2. Understand the concept of a unit rate $\frac{a}{b}$ associated with a ratio $a:b$ with $b \neq 0$, and use rate language in the context of a ratio relationship.

***Example**: This recipe has a ratio of three cups of flour to four cups of sugar, so there is $\frac{3}{4}$ cup of flour for each cup of sugar. We paid $\$75$ for 15 hamburgers, which is a rate of five dollars per hamburger.*

3. Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

    a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.

    b. Solve unit-rate problems, including those involving unit pricing and constant speed.

***Example**: If it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?*

    c. Find a percent of a quantity as a rate per 100; solve problems involving finding the whole, given a part and the percent.

***Example**: 30% of a quantity means $\frac{30}{100}$ times the quantity.*

    d. Use ratio reasoning to convert measurement units within and between measurement systems; manipulate and transform units appropriately when multiplying or dividing quantities.

***Examples**:*

1) Malik is making a recipe, but he cannot find his measuring cups! He has, however, found a tablespoon. His cookbook says that 1 cup = 16 tablespoons. Explain how he could use the tablespoon to measure out the following ingredients: two cups of flour, $\frac{1}{2}$ cup sunflower seeds, and $1\frac{1}{4}$ cup of oatmeal.

2) Jessica is building a doghouse out of wooden planks. If the instructions say the house is 30 inches long, how long would the doghouse be using metric measurements (1 in = 2.54 cm

☐ **6.NS.A. Apply and extend previous understandings of multiplication and division to divide fractions by fractions.**

5. Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem.

***Examples**:*

1) Create a story context for $\frac{2}{3} \div \frac{3}{4}$ and use a visual fraction model to show the quotient.

2) Use the relationship between multiplication and division to explain that $\frac{2}{3} \div \frac{3}{4} = \frac{8}{9}$ because $\frac{3}{4}$ of $\frac{8}{9}$ is $\frac{2}{3}$. In general, $\frac{a}{b} \div \frac{c}{d} = \frac{ad}{bc}$.

3) After hiking $6\frac{1}{2}$ miles along the Salmon River, Fred realized he had traveled $\frac{3}{4}$ of the way to his campsite. What is the total distance Fred will end up traveling during his hike?

4) How many $\frac{3}{4}$ cup servings are in $\frac{2}{3}$ of a cup of yogurt?

5) How wide is a rectangular strip of land with length $\frac{3}{4}$ mi and area $\frac{1}{2}$ square mi?

○ 6.NS.B. Compute fluently with multi-digit numbers and find common factors and multiples.

6. Fluently divide multi-digit numbers using the standard algorithm. **Example:** What is the quotient of 657 and 3 using the standard algorithm?

$$\begin{array}{r} 219 \\ 3\overline{)657} \\ -6 \\ \hline 05 \\ -3 \\ \hline 27 \\ -27 \\ \hline 0 \end{array}$$

7. Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

***Example:** What is the difference of 1.82 and 0.06 using the standard algorithm?*

$$\begin{array}{r} 1.{}^{7}\cancel{8}{}^{1}2 \\ -\,0.\,0\;\;6 \\ \hline 1.\,7\;\;6 \end{array}$$

8. Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common

factor.

Example: *Express* $36 + 8$ *as* $4(9 + 2)$.

△ 6.G.A. Solve real-world and mathematical problems involving area, surface area, and volume.

2. Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = lwh$ and $V = Bh$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.

Seventh Grade Standards: Application of Multiplication and Division

☐ 7.RP.A. Analyze proportional relationships and use them to solve real-world and mathematical problems.

1. Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units.

Example: *If a person walks* $\frac{1}{2}$ *mile in each* $\frac{1}{4}$ *hour, compute the unit rate as the complex fraction* $\frac{1/2}{1/4}$ *miles per hour, equivalently 2 miles per hour.*

2. Recognize and represent proportional relationships between quantities.
    a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
    b. Identify the constant of proportionality in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. Recognize the constant of proportionality as both the unit rate and as the multiplicative comparison between two quantities.
    c. Represent proportional relationships by equations.

Example*: If total cost* t *is proportional to the number* n *of items purchased at a constant price* p*, the relationship between the total cost and the number of items can be expressed as* $t = pn$.

d. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points $(0, 0)$ and $(1, r)$ where r is the unit rate.

3. Use proportional relationships to solve multi-step ratio, rate, and percent problems.

***Examples**: Simple interest, tax, price increases and discounts, gratuities and commissions, fees, percent increase and decrease, percent error*

☐ 7.NS.A. Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

2. Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide integers and other rational numbers.

   d. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $\left(-\frac{1}{2}\right)(-1) = \frac{1}{2}$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.

   e. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with nonzero divisor) is a rational number. If p and q are integers, then $-\left(\frac{p}{q}\right) = \frac{-(p)}{q} = \frac{p}{-(q)}$. Interpret quotients of rational numbers by describing real- world contexts. Interpret quotients of rational numbers by describing real-world contexts.

   f. Apply properties of operations as strategies to multiply and divide rational numbers.

***Example**: $-4(0.25 - 1) = \big((-4) \times 0.25\big) + \big((-4) \times (-1)\big) = -1 + 4 = 3$*

   g. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates or eventually repeats.

3. Solve real-world and mathematical problems involving the four operations with integers and other rational numbers.

***Example**: A water well drilling rig has dug to a depth of -60 feet after one full day of continuous use. If the rig has been running constantly and is currently at a depth of -143.6 feet, for how long has the rig been running? (Modified from Illustrative Mathematics.)*

☐ **7.EE.A. Use properties of operations to generate equivalent expressions.**

3. Apply properties of operations to add, subtract, factor, and expand linear expressions with rational coefficients.

   ***Example**: $4x + 2 = 2(2x + 1)$ and $-3\left(x - \frac{5}{3}\right) = -3x + 5$*

4. Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related.

   ***Examples**:*

   1) $a + 0.05a = 1.05a$ means that "increase by 5%" is the same as "multiply by 1.05."

   2) A shirt at a clothing store is on sale for 20% off the regular price, p. The discount can be expressed as $0.2p$. The new price for the shirt can be expressed as $p - 0.2p$ or $0.8p$.

☐ **7.EE.B. Solve real-life and mathematical problems using numerical and algebraic expressions and equations.**

5. Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (integers, fractions, and decimals). Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.

   ***Examples**:*

   1) If a woman making $\$25$ per hour gets a 10% raise, she will make an additional $\frac{1}{10}$ of her salary an hour, or $\$2.50$, for a new salary of $\$27.50$.

   2) If you want to place a towel bar $9\frac{3}{4}$ inches long in the center of a door that is $27\frac{1}{2}$ inches wide, you will need to place the bar about nine inches from each edge; this estimate can be used as a check on the exact computation.

6. Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

   c. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach.

***Example**: The perimeter of a rectangle is 54 cm. Its width is 6 cm. What is its length?*

d. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem.

***Example**: As a salesperson, you are paid $\$50$ per week plus $\$3$ per sale. This week you want your pay to be at least $\$100$. Write an inequality for the number of sales you need to make, and describe the solutions.*

○ 7.G.A. Draw, construct, and describe geometrical figures and describe the relationships between them.

1. Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

***Example**: Mariko has an $\frac{1}{4}$ inch scale drawing ($\frac{1}{4}$ inch=1 foot) of the floor plan of her house. On the floor plan, the scaled dimensions of her rectangular living room are $4\frac{1}{2}$ inches by $8\frac{3}{4}$ inches. What is the area of her living room in square feet?*

○ 7.G.B. Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

6. Generalize strategies for finding area, volume, and surface areas of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms. Solve real-world and mathematical problems in each of these areas.

***Example**: A playground is being updated. Sand underneath a swing needs to be at least 15 inches deep. The sand under the swings is currently only 12 inches deep. The rectangular area under the swing set measures 9 feet by 12 feet. How much additional sand will be needed to meet the requirement? (Modified from Illustrative Mathematics.)*

Eighth Grade Standards: Application of Multiplication and Division

☐ 8.EE.A. Work with radicals and integer exponents.

9. Know and apply the properties of integer exponents to generate equivalent numerical expressions.

Example: $3^2 \times 3^{-5} = 3^{-3} = \left(\frac{1}{3}\right)^3 = \frac{1}{27}$

10. Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.
11. Use numbers expressed in the form of a single digit multiplied by an integer power of ten (scientific notation) to estimate very large or very small quantities, and express how many times as much one is than the other.

Example*: Estimate the population of the United States as* 3×10^8 *and the population of the world as* 7×10^9*, and determine that the world population is more than* 20 *times larger.*

12. Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities. Interpret scientific notation that has been generated by technology.

☐ 8.EE.B. Understand the connections between proportional relationships, lines, and linear equations.

13. Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.

Example*: Compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.*

○ 8.G.C. Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.

1. Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

RATIOS AND PROPORTIONAL RELATIONSHIPS GRADES 6 – 7

*Note: These standards are also listed as part of the multiplication content progression.

Sixth Grade Standards: Ratios and Proportional Relationships

☐ **6.RP.A. Understand ratio and rate concepts and use ratio and rate reasoning to solve problems.**

1. Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.

   ***Examples**:*

   1) The ratio of wings to beaks in the bird house at the zoo was $2:1$*, because for every two wings there was one beak.*

   2) For every vote candidate A received, candidate C received nearly three votes, meaning that candidate C received approximately three times the number of votes as candidate A, or candidate A received approximately $\frac{1}{3}$ *of the number of votes that candidate C received.*

2. Understand the concept of a unit rate $\frac{a}{b}$ associated with a ratio $a:b$ with $b \neq 0$, and use rate language in the context of a ratio relationship.

   ***Example**: This recipe has a ratio of three cups of flour to four cups of sugar, so there is* $\frac{3}{4}$ *cup of flour for each cup of sugar. We paid* $\$75$ *for* 15 *hamburgers, which is a rate of five dollars per hamburger.*

3. Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

   e. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.

   f. Solve unit-rate problems, including those involving unit pricing and constant speed.

   ***Example**: If it took* 7 *hours to mow* 4 *lawns, then at that rate, how many lawns could be mowed in* 35 *hours? At what rate were lawns being mowed?*

g. Find a percent of a quantity as a rate per 100; solve problems involving finding the whole, given a part and the percent.

***Example**: 30% of a quantity means $\frac{30}{100}$ times the quantity.*

h. Use ratio reasoning to convert measurement units within and between measurement systems; manipulate and transform units appropriately when multiplying or dividing quantities.

***Examples**:*

1) Malik is making a recipe, but he cannot find his measuring cups! He has, however, found a tablespoon. His cookbook says that 1 cup = 16 tablespoons. Explain how he could use the tablespoon to measure out the following ingredients: two cups of flour, $\frac{1}{2}$ cup sunflower seeds, and $1\frac{1}{4}$ cup of oatmeal.

2) Jessica is building a doghouse out of wooden planks. If the instructions say the house is 30 inches long, how long would the doghouse be using metric measurements (1 in = 2.54 cm)?

Seventh Grade Standards: Ratios and Proportional Relationships

☐ 7.RP.A. Analyze proportional relationships and use them to solve real-world and mathematical problems.

4. Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units.

***Example:** If a person walks $\frac{1}{2}$ mile in each $\frac{1}{4}$ hour, compute the unit rate as the complex fraction $\frac{1/2}{1/4}$ miles per hour, equivalently 2 miles per hour.*

5. Recognize and represent proportional relationships between quantities.

e. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

f. Identify the constant of proportionality in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. Recognize the constant

of proportionality as both the unit rate and as the multiplicative comparison between two quantities.

g. Represent proportional relationships by equations.

***Example**: If total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as $t = pn$.*

h. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points $(0, 0)$ and $(1, r)$ where r is the unit rate.

6. Use proportional relationships to solve multi-step ratio, rate, and percent problems.

***Examples**: Simple interest, tax, price increases and discounts, gratuities and commissions, fees, percent increase and decrease, percent error*

MEASUREMENT GRADES K – 5

Kindergarten Standards: Measurement

○ K.MD.A. Describe and compare measurable attributes.

1. Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object.
2. Directly compare two objects with a measurable attribute in common, to see which object has "more of"/ "less of" the attribute, and describe the difference.

   ***Example**: Directly compare the heights of two children and describe one child as taller/shorter.*

First Grade Standards: Measurement

☐ 1.MD.A. Measure lengths indirectly and by iterating (repeating) length units.

1. Order three objects by length; compare the lengths of two objects indirectly by using a third object.
2. Express the length of an object as a whole number of length units by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps.

   Clarification: *Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps. Include use of standard units such as inch-tiles or centimeter tiles.*

○ 1.MD.B. Tell and write time.

3. Tell and write time in hours and half-hours using analog and digital clocks.

Second Grade Standards: Measurement

☐ 2.MD.A. Measure and estimate lengths in standard units.

1. Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.

2. Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen.
3. Estimate lengths using units of inches, feet, centimeters, and meters.
4. Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit.

☐ **2.MD.B. Relate addition and subtraction to length.**

5. Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units.

Clarification: Students may use drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem.

6. Represent whole numbers as lengths from zero on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, ..., and represent whole-number sums and differences within 100 on a number line diagram.

△ **2.MD.C. Work with time and money.**

10. Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m.

△ **2.MD.D. Represent and interpret data.**

1. Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Organize and record data on a line plot (dot plot) where the horizontal scale is marked off in whole-number units.

Third Grade Standards: Measurement

☐ **3.MD.A. Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.**

3. Tell and write time to the nearest minute within the same hour and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes.

Clarification: *Students may use tools such as clocks, number line diagrams, and tables to solve problems involving time intervals.*

2. Identify and use the appropriate tools and units of measurement, both customary and metric, to solve one-step word problems using the four operations involving weight, mass, liquid volume, and capacity (within the same system and unit).

Clarification: *Students may use drawings (such as a beaker with a measurement scale) to represent the problem.*

This standard does not include conversions between units. The focus is on measuring and reasonable estimates, using benchmarks to measure weight, and capacity.

△ 3.MD.B. Represent and interpret data.

4. Generate measurement data by measuring lengths of objects using rulers marked with halves and fourths of an inch. Record and show the data by making a line plot (dot plot), where the horizontal scale is marked off in appropriate units— whole numbers, halves, or fourths.

☐ 3.MD.C. Geometric measurement: Understand concepts of area and relate area to multiplication and to addition.

5. Recognize area as an attribute of plane figures and understand concepts of area measurement.
    a. A square with side length one unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area.
    b. A plane figure which can be covered without gaps or overlaps by n unit squares is said to have an area of n square units.
6. Measure areas by counting unit squares (square cm, square m, square in, square ft, and nonstandard units).
7. Relate area to the operations of multiplication and addition.
    a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.
    b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real-world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.

c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning.

Fourth Grade Standards: Measurement

△ 4.MD.A. Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

1. Know relative sizes of measurement units within any one system of units.
    a. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit.
    b. Record measurement equivalents in a two-column table.

***Example**: Know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs $(1, 12)$, $(2, 24)$, $(3, 36)$, …*

2. Use the four operations to solve word problems involving measurements.
    a. Include problems involving simple fractions or decimals.
    b. Include problems that require expressing measurements given in a larger unit in terms of a smaller unit.
    c. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.

***Clarification**: Measurement may include, but is not limited to, length, area, volume, capacity, mass, weight, and money.*

3. Apply the area and perimeter formulas for rectangles in real-world and mathematical problems.

***Example:** Find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.*

***Clarification:** Students should express their answers in linear (perimeter) and square (area) units. Students are not expected to use the $1\ cm^2$ notation.*

△ 4.MD.B. Represent and interpret data.

4. Make a line plot (dot plot) to show a set of measurements in fractions of a unit $\left(\frac{1}{2}, \frac{1}{4}, \frac{1}{8}\right)$. Solve problems involving addition and subtraction of fractions by using information presented in line plots (dot plots).

Example: *From a line plot (dot plot), find and interpret the difference in length between the longest and shortest specimens in an insect collection.*

○ 4.MD.C. Geometric measurement: Understand concepts of angle and measure angles.

6. Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement.

    a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle.

Example: *An angle that turns through* $\frac{1}{360}$ *of a circle is called a "one-degree angle" and can be used to measure angles.*

    b. An angle that turns through n one-degree angles is said to have an angle measure of n degrees.

6. Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.

7. Solve addition and subtraction problems to find unknown angles on a diagram in real-world and mathematical problems.

    a. Use an equation with a symbol for the unknown angle measure.

    b. Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts.

Fifth Grade Standards: Measurement

△ 5.MD.A. Convert like measurement units within a given measurement system.

1. Convert among different-sized standard measurement units within a given measurement system. Use conversions in solving multi-step, real-world problems.

Example: *Convert* 5 *cm to* 0.05 *m.*

☐ **5.MD.C. Geometric measurement: Understand concepts of volume and relate volume to multiplication and to addition.**

2. Recognize volume as an attribute of solid figures and understand volume measurement in terms of cubic units.
    a. A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume.
    b. A solid figure which can be packed without gaps or overlaps using n unit cubes is said to have a volume of n cubic units.
3. Use concrete and/or visual models to measure the volume of rectangular prisms in cubic units by counting cubic cm, cubic in, cubic ft, and nonstandard units.
4. Relate volume to the operations of multiplication and addition and solve real-world and mathematical problems involving volume.
    a. Find the volume of a right rectangular prism with whole-number edge lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base.

    Example: *To represent the associative property of multiplication,* $(l \times w) \times h = l \times (w \times h)$.

    b. Apply the formulas $V = l \times w \times h$ and $V = B \times h$ (where B stands for the area of the base) for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths, and in the context of solving real-world and mathematical problems.

    c. Recognize volume as additive.
        i. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts.
        ii. Apply this technique to solve real-world problems.

DATA GRADES K – 5

Kindergarten Standards: Data

△ K.MD.B. Classify objects and count the number of objects in each category.

3. Classify objects into given categories; count the numbers of objects in each category (up to and including ten) and sort the categories by count.

First Grade Standards: Data

△ 1.MD.C. Represent and interpret data.

4. Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.

Second Grade Standards: Data

△ 2.MD.D. Represent and interpret data.

2. Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Organize and record data on a line plot (dot plot) where the horizontal scale is marked off in whole-number units.
3. Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems using information presented in the graph.

Third Grade Standards: Data

△ 3.MD.B. Represent and interpret data.

3. Draw a scaled picture graph and scaled bar graph to represent a data set with several categories. Solve one- and two-step “how many more” and “how many less” problems using information presented in scaled bar graphs.

***Example**: Draw a bar graph in which each square in the bar graph might represent five pets.*

4. Generate measurement data by measuring lengths of objects using rulers marked with halves and fourths of an inch. Record and show the data by making a line plot (dot plot), where the horizontal scale is marked off in appropriate units— whole numbers, halves, or fourths.

Fourth Grade Standards: Data

△ 4.MD.B. Represent and interpret data.

4. Make a line plot (dot plot) to show a set of measurements in fractions of a unit $\left(\frac{1}{2}, \frac{1}{4}, \frac{1}{8}\right)$. Solve problems involving addition and subtraction of fractions by using information presented in line plots (dot plots).

***Example:** From a line plot (dot plot), find and interpret the difference in length between the longest and shortest specimens in an insect collection.*

Fifth Grade Standards: Data

△ 5.MD.B. Represent and interpret data.

5. Collect, represent, and interpret numerical data, including whole numbers, and fractional and decimal values.
    a. Interpret numerical data, with whole-number values, represented with tables or line plots.
    b. Use graphic displays of data (line plots (dot plots), tables, etc.) to solve real-world problems using fractional data.

***Example:** Given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.*

STATISTICS AND PROBABILITY GRADES 6 – 8

Sixth Grade Standards: Statistics and Probability

○ 6.SP.A. Develop understanding of statistical variability.

1. Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers.

   Example: *"How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages.*

2. Understand that a set of data collected to answer a statistical question has a distribution, which can be described by its center (median and/or mean), spread (range, interquartile range, and/or mean absolute deviation), and overall shape. The focus of mean absolute deviation (MAD) is visualizing deviations from the mean as a measure of variability as opposed to a focus on calculating MAD.
3. Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.

○ 6.SP.B. Summarize and describe distributions.

4. Display numerical data in plots on a number line, including dot plots, histograms, and box plots.
5. Summarize numerical data sets in relation to their context, such as by:
   a. Reporting the number of observations.
   b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.
   c. Giving quantitative measures of center (median, and/or mean) and variability (range, interquartile range, and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.
   d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.

Examples: *Bobbie is a sixth grader who competes in the* 100 *meter hurdles. In eight track meets during the season, she recorded the following times (to the nearest one hundredth of a second).*

$18.11, 31.23, 17.99, 18.25, 17.50, 35.55, 17.44, 17.85$

Is the mean or the median a better representation of Bobbie's hurdle time? Justify your answer. (From Illustrative Mathematics.)

Seventh Grade Standards: Statistics and Probability

△ 7.SP.A. Use random sampling to draw inferences about a population.

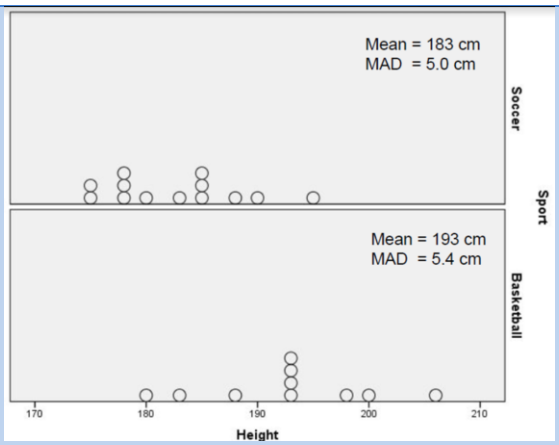
1. Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.
2. Use data from a random sample about an unknown characteristic of a population. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions, i.e., generate a sampling distribution.

***Example**: Estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.*

○ 7.SP.B. Draw informal comparative inferences about two populations.

3. Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability.

***Example**: The difference in the mean height between players on the basketball team versus the soccer team is 10 cm. This difference in the means - 10 cm - is about twice the variability (mean absolute deviation) on either team (i.e., mean divided by the MAD). On a dot plot, the separation between the two distributions of heights is noticeable.*



4. Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations.

   ***Example**: Decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.*

△ 7.SP.C. Investigate chance processes and develop, use, and evaluate probability models.

5. Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. A probability near 0 indicates an unlikely event, a probability around $\frac{1}{2}$ indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.

   ***Example**: The likelihood of drawing a heart from a deck of cards is 0.25. The likelihood of flipping a coin and landing on heads is 0.5. It is more likely that a flipped coin will land on heads than it is to choose a heart from a deck of cards. (0.5 is greater than 0.25).*

6. Approximate the (theoretical) probability of a chance event by collecting data and observing its long-run relative frequency (experimental probability). Predict the approximate relative frequency given the (theoretical) probability.

   ***Examples**:*

   1) When drawing chips out of a bag containing an unknown number of red and white chips, estimate the probability of selecting a particular chip color given 50 draws.

   2) When rolling a number cube 600 times, predict that a 3 or 6 would be rolled approximately 200 times, but probably not exactly 200 times.

7. Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.

a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events.

***Example**: If a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.*

b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process.

***Example**: Find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?*

8. Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.

a. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.

b. Represent sample spaces for compound events using methods such as organized lists, tables, and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event.

c. Design and use a simulation to generate frequencies for compound events.

***Example**: Use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?*

Eighth Grade Standards: Statistics and Probability

△ 8.SP.A. Investigate patterns of association in bivariate data.

1. Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.
2. Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.

3. Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept.

   ***Example**: In a linear model for a biology experiment, interpret a slope of* 1.5 *cm/hr as meaning that an additional hour of sunlight each day is associated with an additional* 1.5 *cm in mature plant height.*

4. Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables.

   ***Example**: Collect data from students in your school on grade level (sixth, seventh, and eighth) and whether or not they have assigned chores at home (yes, no). Is there evidence that a particular grade level tends to have chores? (In this example the two variables are grade level and chores.)*

Pages 157 through 164 in the Idaho Content Standards for Mathematics show how the Data and Statistics and Probability Domain extends into Grades 9 through 12.

GEOMETRY GRADES K – 8

Kindergarten Standards: Geometry

○ K.G.A. Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).

1. Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as “above,” “below,” beside, “in front of,” “behind,” and “next to.”
2. Correctly name shapes regardless of their orientations or overall size.
3. Identify shapes as two-dimensional (lying in a plane, “flat”) or three-dimensional (“solid”).

△ K.G.B. Analyze, compare, create, and compose shapes.

4. Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts, and other attributes.

   ***Examples**:*

   1) Number of sides and vertices/ “corners”

   2) Having sides of equal length

5. Model shapes in the world by building shapes from components/materials and drawing shapes.

   ***Clarification:** Components/materials may include: sticks, clay balls, marshmallows and/or spaghetti.*

6. Compose simple shapes to form larger two-dimensional shapes.

   ***Example:** Can you join these two triangles with full sides touching to make a rectangle?*

First Grade Standards: Geometry

○ 1.G.A. Reason with shapes and their attributes.

1. Compare defining attributes and non-defining attributes of two- and three-dimensional shapes; build and draw shapes that possess defining attributes.

Clarification: *The defining attributes of triangles are closed and three-sided versus non-defining attributes of color, orientation, and overall size.*

2. Compose two-dimensional (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape.

Clarification: *Students do not need to learn formal names such as "right rectangular prism."*

3. Partition circles and rectangles into two and four equal shares. Understand for these examples that decomposing into more equal shares creates smaller shares.
    a. Describe the shares using the words "halves," "fourths," and "quarters," and use the phrases "half of," "a fourth of," and "a quarter of."
    b. Describe the whole as two of, or four of, the shares.

Second Grade Standards: Geometry

○ 2.G.A. Reason with shapes and their attributes.

2. Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces. Identify triangles, squares, rectangles, rhombi, trapezoids, pentagons, hexagons, octagons, and cubes.

2. Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.

3. Partition circles and rectangles into two, three, or four equal shares. Understand for these examples that decomposing into more equal shares creates smaller shares.
    a. Describe the shares using the words "halves," "thirds," "fourths," and "quarter," and use the phrases "half of," "a third of," "a fourth of," and "quarter of."
    b. Describe the whole as two of, three of, or four of the shares.
    c. Recognize that equal shares of identical wholes need not have the same shape.

Third Grade Standards: Geometry

△ 3.G.A. Reason with shapes and their attributes.

1. Understand that shapes in different categories may share attributes, and that the shared attributes can define a larger category. Compare and classify shapes by their sides and angles. Recognize rhombi, rectangles, squares, and trapezoids as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.
2. Partition two-dimensional figures into equal areas, and express the area of each part as a unit fraction of the whole.

   ***Example**: Draw lines to separate a shape into 4 parts with equal area, and describe the area of each part as $\frac{1}{4}$ of the area of the shape.*

☐ 3.MD.C. Geometric measurement: Understand concepts of area and relate area to multiplication and to addition.

5. Recognize area as an attribute of plane figures and understand concepts of area measurement.
   a. A square with side length one unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area.
   b. A plane figure which can be covered without gaps or overlaps by n unit squares is said to have an area of n square units.
6. Measure areas by counting unit squares (square cm, square m, square in, square ft, and nonstandard units).

Fourth Grade Standards: Geometry

○ 4.MD.C. Geometric measurement: Understand concepts of angle and measure angles.

7. Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement.
   c. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle.

Example: *An angle that turns through $\frac{1}{360}$ of a circle is called a "one-degree angle" and can be used to measure angles.*

d. An angle that turns through n one-degree angles is said to have an angle measure of n degrees.

7. Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.

7. Solve addition and subtraction problems to find unknown angles on a diagram in real-world and mathematical problems.

   a. Use an equation with a symbol for the unknown angle measure.

   b. Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts.

○ 4.G.A. Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

1. Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.

2. Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.

3. Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.

Fifth Grade Standards: Geometry

○ 5.G.A. Graph points on the coordinate plane to solve real-world and mathematical problems.

1. Describe and understand the key attributes of the coordinate plane.

   a. Use a pair of perpendicular number lines (axes) with the intersection of the lines (the origin $(0,0)$) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates.

b. Understand that the x-coordinate, the first number in an ordered pair, indicates movement parallel to the x-axis starting at the origin; and the y-coordinate, the second number, indicates movement parallel to the y-axis starting at the origin.

2. Represent real-world and mathematical problems by graphing points in the first quadrant of the coordinate plane (x and y both have positive values), and interpret coordinate values of points in the context of the situation.

○ 5.G.B. Classify two-dimensional figures into categories based on their properties.

3. Understand that attributes belonging to a category of two-dimensional figures also belong to all of the subcategories of that category.

   Example: *All rectangles have four right angles and squares are rectangles, so all squares have four right angles.*

4. Classify two-dimensional figures in a hierarchy based on properties.

   Example: *All rectangles are parallelograms because they are all quadrilaterals with two pairs of opposite sides parallel.*

Sixth Grade Standards: Geometry

△ 6.G.A. Solve real-world and mathematical problems involving area, surface area, and volume.

1. Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

2. Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = lwh$ and $V = Bh$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.

3. Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side and area by joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.

4. Represent three-dimensional figures using nets made up of rectangles and triangles and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

Example: *Explain how you could find the surface area of a rectangular prism given a three-dimensional representation (Fig. A) or a net (Fig. B).*

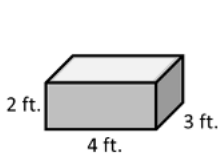


Fig. A

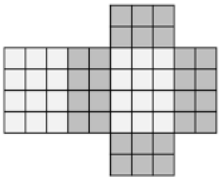


Fig. B

Seventh Grade Standards: Geometry

○ 7.G.A. Draw, construct, and describe geometrical figures and describe the relationships between them.

2. Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

Example*: Mariko has an* $\frac{1}{4}$ *inch scale drawing (*$\frac{1}{4}$ *inch=*1 *foot) of the floor plan of her house. On the floor plan, the scaled dimensions of her rectangular living room are* $4\frac{1}{2}$ *inches by* $8\frac{3}{4}$ *inches. What is the area of her living room in square feet?*

3. Draw (freehand, with ruler and protractor, and with technology) two-dimensional geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine unique triangles, more than one triangle, or no triangle.

Example*: A triangle with side lengths* 3 *cm,* 4 *cm, and* 5 *cm exists. Use a compass and ruler to draw a triangle with these side lengths. (Modified from Engage NY M6L9.)*

4. Describe the shape of the two-dimensional face of the figure that results from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.

○ 7.G.B. Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

4. Understand the attributes and measurements of circles.
    a. Know that a circle is a two-dimensional shape created by connecting all of the points equidistant from a fixed point called the center of the circle.
    b. Develop an understanding of circle attributes including radius, diameter, circumference, and area and investigate the relationships between each.
    c. Informally derive and know the formulas for the area and circumference of a circle and use them to solve problems.
5. Use facts about supplementary, complementary, vertical, and adjacent angles to write equations and use them to solve for an unknown angle in a figure.

    ***Example**: The ratio of the measurement of an angle to its complement is* $1:2$*. Create and solve an equation to find the measurement of the angle and its complement. (Modified from Engage NY M5L1.)*
6. Generalize strategies for finding area, volume, and surface areas of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms. Solve real-world and mathematical problems in each of these areas.

    ***Example**: A playground is being updated. Sand underneath a swing needs to be at least* 15 *inches deep. The sand under the swings is currently only* 12 *inches deep. The rectangular area under the swing set measures* 9 *feet by* 12 *feet. How much additional sand will be needed to meet the requirement? (Modified from Illustrative Mathematics.)*

Eighth Grade Standards: Geometry

☐ 8.G.A. Understand congruence and similarity using physical models, transparencies, or geometry software.

1. Verify experimentally the properties of rotations, reflections, and translations:
    a. Lines are transformed to lines, and line segments to line segments of the same length.
    b. Angles are transformed to angles of the same measure.
    c. Parallel lines are transformed to parallel lines.

2. Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations.

   ***Example**: Given two congruent figures, describe a sequence that exhibits the congruence between them.*

3. Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

   ***Example**: The image of Triangle ABC with $A = (-3, 0)$, $B = (-3, -2)$, and $C = (4, -2)$ would have coordinates $A' = (-3 - 3, 0 + 2) = (-6, 2)$, $B' = (-3 - 3, -2 + 2) = (-6,0)$, and $C' = (4 - 3, -2 + 2) = (1,0)$ following a translation 3 units to the left and 2 units up.*

4. Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations.

   ***Example**: Given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.*

5. Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.

   ***Example**: Arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*

☐ **8.G.B. Understand and apply the Pythagorean Theorem.**

6. Analyze and justify the Pythagorean Theorem and its converse using pictures, diagrams, narratives, or models.
7. Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.
8. Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

○ **8.G.C. Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.**

9. Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

Pages 148 through 156 in the Idaho Content Standards for Mathematics show how the Geometry Domain extends into Grades 9 through 12.