

IDAHO CONTENT STANDARDS FOR MATHEMATICS

9-12 Mathematics Course Planning Guide



IDAHO DEPARTMENT OF EDUCATION
CONTENT AND CURRICULUM | MATHEMATICS

650 W STATE STREET, 2ND FLOOR
BOISE, IDAHO 83702
208 332 6800 OFFICE / 711 TRS
WWW.SDE.IDAHO.GOV

CREATED 02/28/2024

TABLE OF CONTENTS

Introduction 3

Recommendations For Districts..... 4

Foundational Standards 7

 Number and Quantity (N) 8

 Algebra (A)..... 9

 Functions (F) 12

 Geometry (G)..... 15

 Statistics and Probability (S)..... 19

Advanced Standards by Career Pathway 21

 Number and Quantity (N) 21

 Algebra (A)..... 22

 Functions (F) 25

 Geometry (G)..... 26

 Statistics and Probability (S)..... 27

College Standards 29

 Number and Quantity (N) 29

 Algebra (A)..... 32

 Functions (F) 35

 Geometry (G)..... 42

 Statistics and Probability (S)..... 44

References 49

Additional Recommended reading..... 49

Appendix A: STANDARDS TO ISAT ALIGNMENT 50

 Grade 11 ISAT Target Summary for Claim 1..... 52

 Grade 11 ISAT Target Summary for Claims 2 through 4 54

INTRODUCTION

The 2022 Idaho Content Standards for Mathematics support a progression of increasing knowledge and skills. For grades nine through twelve, content is organized into five conceptual categories:

- Number and Quantity (N)
- Algebra (A)
- Functions (F)
- Geometry (G)
- Statistics and Probability (S)

While the standards document clearly articulates rigorous standards for high school mathematics, it does not provide guidance for districts to assign standards to grade levels or courses. Because Idaho is a local control state, assigning standards to courses at the high school level is a local district responsibility.

This document was created as a resource for districts to assist them in assigning standards to high school mathematics courses. These recommendations are based on a vision for high school mathematics programs articulated by the Idaho Math Transitions Steering Committee, shown in Figure 1. This committee is comprised of a mathematics professor from each public college and university as well as a K-12 teacher or leader for each of Idaho's six geographical regions. The steering committee worked with Regional Math Specialists from Idaho's Regional Math Centers, college math faculty, and high school educators to organize the Idaho Content Standards for Mathematics into recommended grade bands. The standards were compared to the content taught in the three general education mathematics courses that all of Idaho's public colleges and universities offer. These courses are:

MATH 123: Math in the Modern Society

MATH 143: Precalculus I: Algebra

MATH 153: Statistical Reasoning

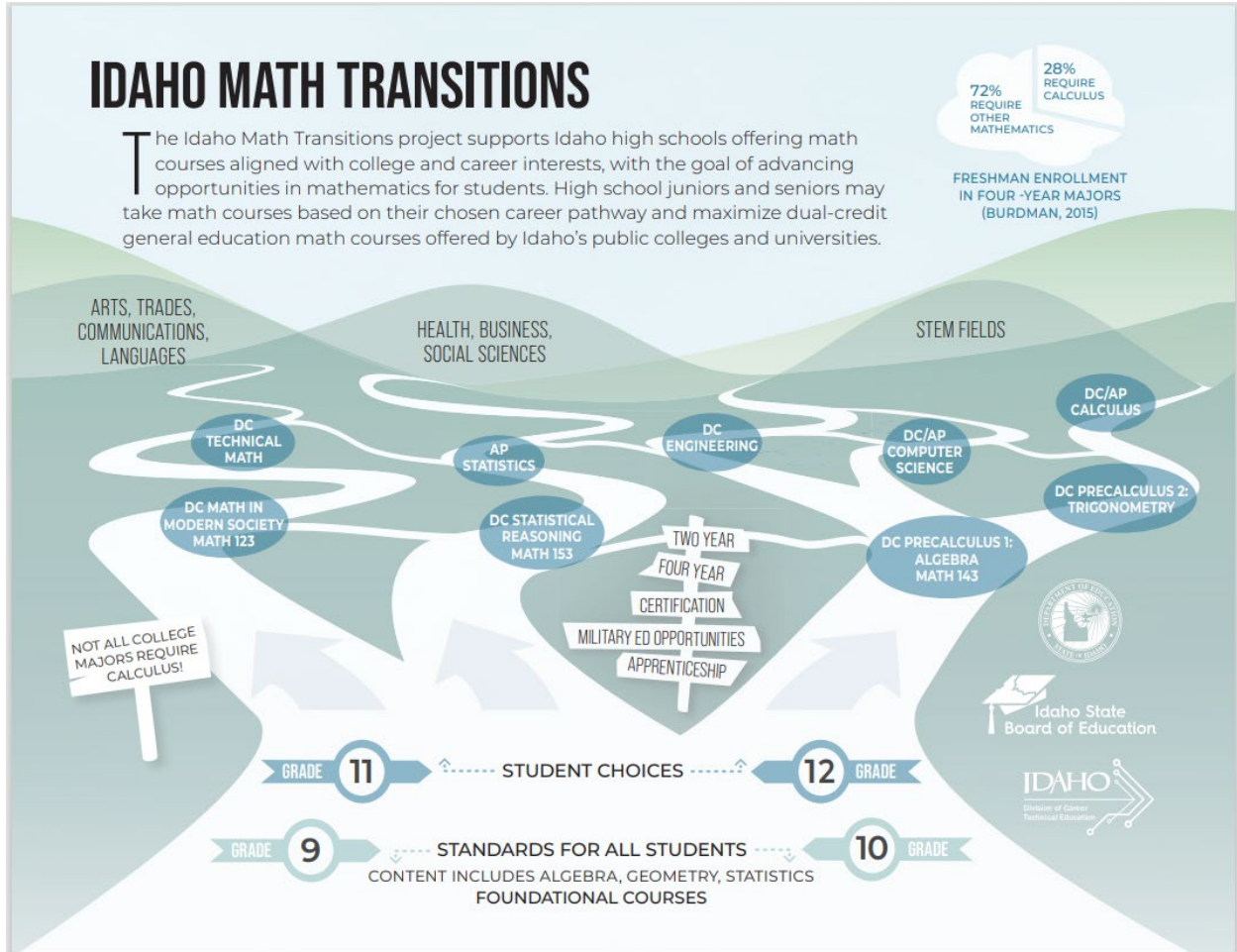
These three general education courses are required for different career pathways at Idaho's public colleges and universities. By aligning high school mathematics content standards to the content taught in courses that can be taken for dual credit through Idaho's Advanced Opportunities program, we hope to ensure a successful transition in mathematics from high school to college for all Idaho high school graduates.

The standards are organized into three types of standards:

- **Foundational Standards** are mathematics standards that all high school graduates need to master in order to be successful in both college and careers.
- **Advanced Standards** are standards that are considered pre-requisite standards needed for one or more of the three college pathways.

- College Standards are taught at the college level through dual-credit courses or in more advanced college mathematics courses.

Figure 1: Idaho Math Transitions Graphic



RECOMMENDATIONS FOR DISTRICTS

The Idaho Math Transitions Steering Committee provides recommendations for Idaho high schools to create a successful transition in mathematics from high school to college. These recommendations are based on research on college success in mathematics (see references section), and data on retention and failure rates of mathematics courses typically taken by freshman at Idaho’s colleges and universities.

Recommendations for Idaho high school mathematics programs:

- Strongly encourage all students to complete four years of mathematics courses in high school. Students who complete four years of mathematics in high school are more

successful in college mathematics courses than students who do not have four years of mathematics (Zelkowski, 2011). Math courses taken in high school have a significant effect on whether a student goes to college by age 21 (Aughinbaugh, 2012; Jia, 2021). Advanced high school mathematics courses that include statistics and applied mathematics can engage students in a broad understanding of mathematical sciences used in modern industries (Son & Stigler, 2023).

- Students desiring to continue on to college in a Science, Technology, Engineering or Mathematics (STEM) related major should have four years of mathematics courses in high school, including Calculus when possible. Studies show that the most successful college freshman in Calculus courses have taken Calculus both as a high school senior and as a college freshman (Bressoud, 2016). Students who take Calculus in high school who are majoring in a STEM field may be asked to take Calculus again at the college level depending on their placement data. Students can also take Calculus for the first time in college and be very successful as a STEM major. Calculus completion in high school is not a prerequisite for college admission as a STEM major.
- Students pursuing a non-STEM related major may not need Calculus for their chosen degree. Students can broaden their mathematical understanding through other mathematics courses such as Algebra 2 and Algebra 2 equivalent courses including trigonometry, statistics, data science, computer science, engineering, business math, quantitative reasoning and applied math. Being confident and successful in mathematics, seeing mathematics as relevant, and having a strong foundation in problem solving may be more important than taking Calculus before graduating from high school.
- Align junior and senior year courses to general education mathematics courses offered at all of Idaho's public colleges and universities and in many Idaho high schools as dual credit courses. Consider creating access to these general education mathematics courses for high school students by working with [Idaho Digital Learning Alliance](#) (IDLA) or local colleges and universities. Juniors and seniors can take any of these courses and complete their college general education math course prior to their freshman year of college with Idaho's [Advanced Opportunities](#) program. The recommended college mathematics courses for high school students are:
 - Precalculus I: Algebra and Precalculus 2: Trigonometry – for all STEM fields
 - Math in the Modern Society – for Communications, Trades, Arts and Languages
 - Statistical Reasoning – for Health, Business, and Social Sciences
- High Schools can create mathematics courses for juniors and seniors centered around college and career pathways. See The [Launch Years Report](#) published by the University of Texas at Austin Charles A. Dana Center and Sparks (2018) for additional guidance on designing high school mathematics programs.

- Engage students in the Standards for Mathematical Practice described in the Idaho Content Standards through the use of high-impact teaching practices. The Standards for Mathematical Practice (SMP) describe the type of mathematical thinking needed for the for students to be ready for a world of STEM. The Standards for Mathematical Practice are:

SMP1: Make sense of problems and persevere in solving them.

SMP2: Reason abstractly and quantitatively.

SMP3: Construct viable arguments and critique the reasoning of others.

SMP4: Model with mathematics.

SMP5: SMP1: Use appropriate tools strategically.

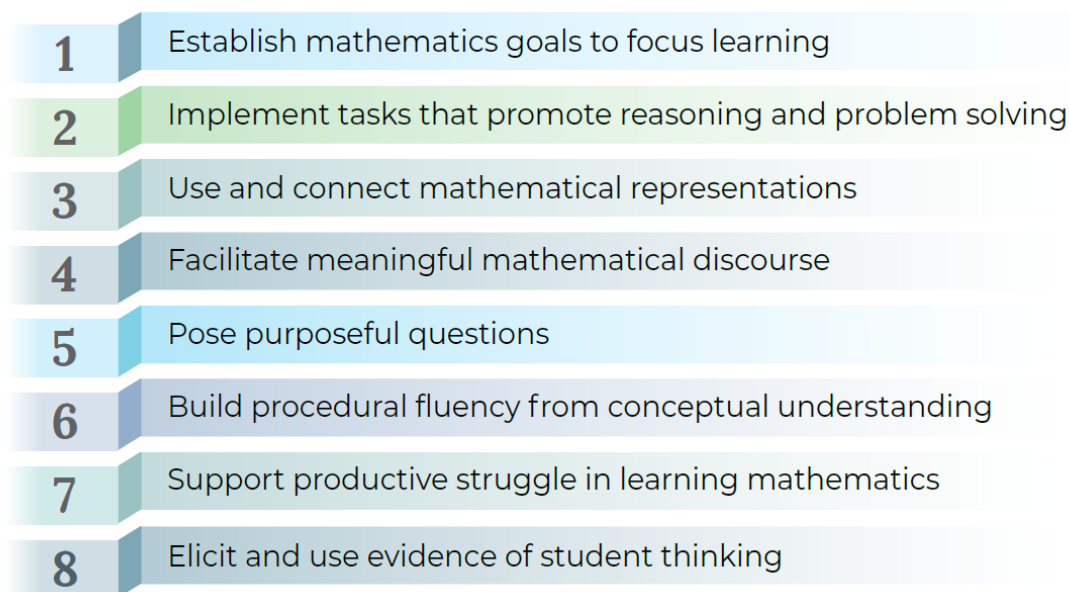
SMP6: Attend to precision.

SMP7: Look for and make use of structure.

SMP8: Look for and express regularity in repeated reasoning.

- Design lessons using the practices listed in the Idaho Math Instructional Framework. These practices are supported by research summarized by the National Council Teachers of Mathematics in *Principles to Actions: Ensuring Mathematical Success for All* (NCTM, 2014). Figure 2 shows these eight high-impact teaching practices.

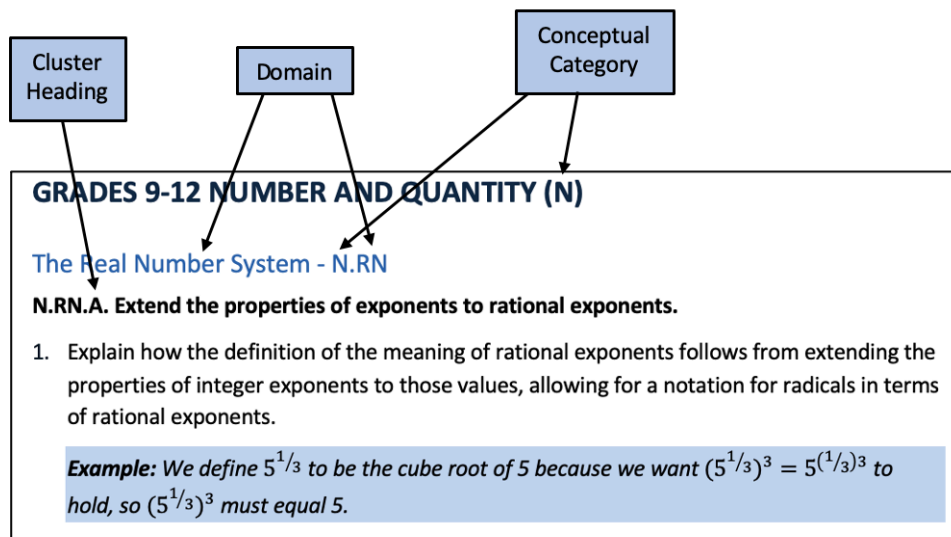
Figure 2: Idaho Mathematics Instructional Framework



FORMATTING NOTES

- Throughout this document, standards listed with a ★ are standards that are opportunities to engage students in mathematical modeling. See pages 145 and 146 in the 2022 Idaho Content Standards for Mathematics for a thorough description of mathematical modeling.
- The coding for the standards in this document is consistent with the 2022 Idaho Content Standards for Mathematics. For each 9 – 12 conceptual category, the standard number begins with the identifier for the conceptual category code (N, A, F, G, S), followed by the domain code, and the standard number, as shown in Figure 3. In this document, the conceptual categories, the domain headings and the cluster headings are used to organize standards as foundational, advanced, or college standards. Within the original standards clusters, standards have been identified as different levels of difficulty. For this reason, the numbering within the tables in this document is not sequential so that the numbering continues to match the original standards document.

Figure 2: Idaho Content Standards for Mathematics Coding System



A Standards Overview Chart is provided in Appendix A as a summary of the designation for each 9-12 Idaho Content Standard for Mathematics.

FOUNDATIONAL STANDARDS

These standards are foundational for all students regardless of career pathway. The majority of the Grade 11 Idaho Standards for Achievement Test (ISAT) will assess these standards (See Appendix B). Some of the Grade 11 ISAT will assess content more advanced than these foundational standards, so these standards should not be the stopping point for most students in their high school mathematics education. Appendix B provides details on the alignment of each category of standards to the ISAT.

Number and Quantity (N)

The Real Number System - N.RN

N.RN.A. Extend the properties of exponents to rational exponents.

1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.

***Example:** We define $5^{\frac{1}{3}}$ to be the cube root of 5 because we want $(5^{\frac{1}{3}})^3 = 5^{(\frac{1}{3})^3}$ to hold, so $(5^{\frac{1}{3}})^3$ must equal 5.*

2. Rewrite expressions involving radicals and rational exponents using the properties of exponents.

***Example:** Solving the volume of a cube formula, $V = s^3$, for s would involve rewriting the solution as either $s = \sqrt[3]{V}$ or $s = V^{\frac{1}{3}}$.*

N.RN.B. Use properties of rational and irrational numbers.

3. Explain why the sum and product of two rational numbers is rational; why the sum of a rational number and an irrational number is irrational; and why the product of a nonzero rational number and an irrational number is irrational.

Quantities – N.Q

N.Q.A. Reason quantitatively and use units to solve problems.

1. Use units as a way to understand problems and to guide the solution of multi-step problems choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.★

2. Define appropriate quantities for the purpose of descriptive modeling.★

Algebra (A)

Seeing Structure in Expressions – A.SSE

A.SSE.A. Interpret the structure of linear, quadratic, exponential, polynomial, and rational expressions.

1. Interpret expressions that represent a quantity in terms of its context.★
 - a. Interpret parts of an expression, such as terms, factors and coefficients.
 - b. Interpret complicated expressions by viewing one or more of their parts as a single entity.

Example: Interpret $P(1 + r)^n$ as the product of P and a factor not depending on P .

2. Use the structure of an expression to identify ways to rewrite it.

Example: See $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.

A.SSE.B. Write expressions in equivalent forms to solve problems.

3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
 - a. Factor a quadratic expression to reveal the zeros of the function it defines.
 - c. Use the properties of exponents to transform expressions for exponential functions.

Example: The expression 1.15^t can be rewritten as $(1.15^{\frac{1}{12}})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.

Arithmetic with Polynomials and Rational Expressions – A.APR

A.APR.A. Perform arithmetic operations on polynomials.

1. Demonstrate understanding that polynomials form a system analogous to the integers; namely, they are closed under certain operations.
 - a. Perform operations on polynomial expressions (+, -, \times , /) and compare the system of polynomials to the system of integers when performing operations.

Teacher Note:

Teacher Note: Division with monomials is foundational, but division with polynomials is an Advanced Standard in preparation for Math 143 Precalculus 1.

- b. Factor and/or expand polynomial expressions, identify and combine like terms, and apply the distributive property.

Teacher Note:

Factoring and/or expanding polynomial expressions is an Advanced Standard in preparation for Math 143 Precalculus 1: Algebra. Identifying and combining like terms and applying the distributive property is foundational.

Creating Equations – A.CED

A.CED.A. Create equations that describe numbers or relationships.

1. Create one-variable equations and inequalities to solve problems, including linear, quadratic, rational, and exponential functions.

Example: Four people may be seated at one rectangular table. If two rectangular tables are placed together end-to-end, six people may be seated at the table. If ten tables are placed together end-to-end, how many people can be seated? How many tables are needed for n people?

Teacher Note:

Focus on creating linear equations and inequalities to solve problems at the foundational level. This standard will be expanded upon at the advanced and college levels.

2. Interpret the relationship between two or more quantities.★
 - a. Define variables to represent the quantities and write equations to show the relationship.★

Example: The cost of parking in the parking garage is \$2.00 for the first hour and \$1.00 for every hour after that. Write an equation in terms of x and y that shows the total cost for parking, y , for x hours. Use the equation to calculate the cost for parking in the garage for 5 hours.

- b. Use graphs to show a visual representation of the relationship while adhering to appropriate labels and scales.★

Example: Using the equation from **A.CED.A.2.a**, show how the graph of the equation can be used to predict the cost for a specified amount of time.

3. Represent constraints using equations or inequalities and interpret solutions as viable or non-viable options in a modeling context.★
4. Represent constraints using systems of equations and/or inequalities and interpret solutions as viable or non-viable options in a modeling context.★
5. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.★

Example: Rearrange Ohm's law $V = IR$ to highlight resistance R .

Reasoning with Equations and Inequalities – A.REI

A.REI.A. Understand solving equations as a process of reasoning and explain the reasoning.

1. Explain each step in solving a simple equations as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify or refute a solution method.

A.REI.B. Solve equations and inequalities in one variable.

Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

A.REI.C. Solve systems of equations.

6. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

Example: A school club is selling hats and t-shirts for a fundraiser. The group expects to sell a total of 50 items. They make a profit of 15 dollars for each t-shirt sold and 5 dollars for each hat sold. How many hats and t-shirts will the school club need to sell to make a profit of \$300?

A.REI.D. Represent and solve equations and inequalities graphically.

10. Demonstrate understanding that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane. Show that any point on the graph of an equation in two variables is a solution to the equation.

Functions (F)

Interpreting Functions – F.IF

F.IF.A. Understand the concept of a function and use function notation.

1. Demonstrate understanding that a function is a correspondence from one set (called the domain) to another set (called the range) that assigns to each element of the domain exactly one element of the range: If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x . The graph of f is the graph of the equation $y = f(x)$.
2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

Example: Given a function representing a car loan, determine the balance of the loan at different points in time.

F.IF.B. Interpret functions that arise in applications in terms of the context. Include linear, quadratic, exponential, rational, polynomial, square root and cube root, trigonometric, and logarithmic functions.

Teacher Note:

Foundational standards primarily emphasize linear functions. Foundational standards also include a less formal exploration of the basic attributes of exponential and quadratic functions.

The further reinforcement of these standards (F.IF.B.4 and F.IF.B.5) as prerequisite concepts for the named dual-credit courses would be for continued exploration of exponential and quadratic functions as well as more complex functions such as cubic and logarithmic functions.

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maxima and minima; symmetries; end behavior; and periodicity.★

Example: Given a context or verbal description of a relationship, sketch a graph that models the context or description and shows its key features.

Teacher Note:

Foundational standards primarily emphasize linear functions, but interpreting exponential and quadratic functions is also relevant for F.IF.B.4. Interpreting other functions is related to

advanced standards.

5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.★

Example: *If the function $h(n)$ gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*

Teacher Note:

Foundational standards primarily emphasize linear functions, though interpreting domain and range from various graphs is appropriate for foundational standards. Other functions are related to advanced standards.

6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★

Teacher Note:

F.IF.B.6 should be focused primarily on linear functions as a foundational standard.

F.IF.C. Analyze functions using different representations.

7. Graph functions expressed symbolically and show key features of the graphs, by hand in simple cases and using technology for more complicated cases.
 - a. Graph linear and quadratic functions and show intercepts, maxima, and minima.★
8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables or by verbal descriptions).

Example: *Given a graph of one polynomial function and an algebraic expression for another, say which has the larger/smaller relative maximum and/or minimum.*

Building Functions – F.BF

F.BF.A. Build a function that models a relationship between two quantities.

1. Write a function that describes a relationship between two quantities. Functions could include linear, exponential, quadratic, simple rational, radical, logarithmic, and trigonometric.★

Teacher Note:

Specifically, linear and exponential functions are relevant to the foundational standards portion of F.BF.A. Other functions are addressed in the advanced standards.

- a. Determine an explicit expression, a recursive process, or steps for calculation from a context.★

Linear, Quadratic, and Exponential Models – F.LE

Teacher Note:

Foundational standards primarily emphasize linear functions. Foundational standards also include a less formal exploration of the basic attributes of exponential and quadratic functions.

The further reinforcement of these ideas as prerequisite concepts for the named dual-credit courses in the Advanced Standards section would be for continued exploration of exponential and quadratic functions.

F.LE.A. Construct and compare linear, quadratic, and exponential models and solve problems.

1. Distinguish between situations that can be modeled with linear functions and with exponential functions.★
 - a. Demonstrate that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.★
 - b. Identify situations in which one quantity changes at a constant rate per unit interval relative to another.★
 - c. Identify situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.★
2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of the relationship, or two input-output pairs (including reading these from a table).★

F.LE.B. Interpret expressions for functions in terms of the situation they model.

5. Interpret the parameters in a linear or exponential function (of the form $f(x) = b^x + k$) in terms of a context.★

Geometry (G)

Congruence – G.CO

G.CO.A. Experiment with transformations in the plane.

1. Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
2. Represent transformations in the plane and describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not.

Example: Translation versus horizontal stretch.

3. Describe the rotations and reflections that carry a given figure (rectangle, parallelogram, trapezoid, or regular polygon) onto itself.
4. Develop definitions of rotations, reflections and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.
5. Draw the transformation (rotation, reflection, or translation) for a given geometric figure.

Example: Given quadrilateral $TMEJ$ with vertices $T(0, -1)$, $M(3, -2)$, $E(-1, -5)$, and $J(-3, -2)$, reflect the shape across the x -axis.

Teacher Note: Emphasis should be placed on spatial reasoning over procedures with coordinates.

6. Specify a sequence of transformations that will carry a given figure onto another.

G.CO.B. Understand congruence in terms of rigid motions.

7. Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.
8. Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
9. Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

Example: In $\triangle ABC$ and $\triangle ABD$ (with shared side \underline{AB}), we are given that $\angle BAC \cong \angle BAD$ and $\angle ABC \cong \angle ABD$. What pair(s) of corresponding parts is/are needed to ensure the triangles are congruent by either ASA, SAS, or SSS? What rigid motion would show the triangles are congruent?

G.CO.C. Prove geometric theorems and, when appropriate, the converse of theorems.

Teacher Note:

The value of proof in these contexts is a transferable understanding of deductive reasoning and argumentation; it is more about the argumentation than the specific geometric topic.

10. Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent, and conversely prove lines are parallel; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.
11. Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180 degrees; base angles of isosceles triangles are congruent, and conversely prove a triangle is isosceles; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.
12. Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.
 - a. Prove theorems about polygons. Theorems include: the measures of interior and exterior angles; apply properties of polygons to the solutions of mathematical and contextual problems.

G.CO.D. Make geometric constructions.

13. Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Constructions include: copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.
14. Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

Similarity, Right Triangles, and Trigonometry – G.SRT

G.SRT.A. Understand similarity in terms of similarity transformations.

1. Verify experimentally the properties of dilations given by a center and a scale factor.
 - a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
 - b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.

2. Use the definition of similarity to decide if two given figures are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.
3. Use the properties of similarity transformations to establish the Angle-Angle (AA) criterion for two triangles to be similar.

Example: Given $\triangle ABC$ and $\triangle DEF$, $\angle A \cong \angle D$, and $\angle B \cong \angle E$, show that $\triangle ABC \sim \triangle DEF$ using a sequence of translations, rotations, reflections, and/or dilations.

G.SRT.B. Prove theorems involving similarity.

5. Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

G.SRT.C. Define trigonometric ratios and solve problems involving right triangles.

6. Demonstrate understanding that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute triangles

Circles – G.C

G.C.A. Understand and apply theorems about circles.

2. Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.

Expressing Geometric Properties with Equations – G.GPE

G.GPE.B. Use coordinates to prove simple geometric theorems algebraically.

4. Use coordinates to prove simple geometric theorems algebraically, including the distance formula and its relationship to the Pythagorean Theorem.

Example: Prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point $(1, \sqrt{3})$ lies on the circle centered at the origin and containing the point $(0, 2)$.

5. Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems.

Example: Find the equation of a line parallel or perpendicular to a given line that passes through a given point.

6. Find the point on a directed line segment between two given points that partitions the segment in a given ratio.
7. Use coordinates to compute perimeters of polygons and areas of triangles and rectangles (e.g., using the distance formula).★

Geometric Measurement and Dimension – G.GMD

G.GMD.A. Explain volume formulas and use them to solve problems.

1. Give an informal argument for the formulas for the circumference of a circle; area of a circle; volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments.
3. Use volume formulas for cylinders, pyramids, cones and spheres to solve problems.★

Example: *The tank at the top of the Meridian Water Tower is roughly spherical. If the diameter of the sphere is 50.35 feet, approximately how much water can the tank hold?*

Modeling with Geometry – G.MG

G.MG.A. Apply geometric concepts in modeling situations.

1. Use geometric shapes, their measures, and their properties to describe objects.★

Example: *Modeling a tree trunk or a human torso as a cylinder.*

2. Apply concepts of density based on area and volume in modeling situations.★

Example: *Persons per square mile, BTUs per cubic foot*

Teacher Note:

Understanding of this standard is particularly important for students interested in pursuing STEM.

3. Apply geometric methods to solve design problems.★

Example: *Designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios.*

4. Use dimensional analysis for unit conversions to confirm that expressions and equations make sense.★

Statistics and Probability (S)

Interpreting Categorical and Quantitative Data – S.ID

S.ID.A. Summarize, represent, and interpret data on a single count or measurement variable. Use calculators, spreadsheets, and other technology as appropriate.

1. Differentiate between count data and measurement variable.★
2. Represent measurement data with plots on the real number line (dot plots, histograms, and box plots).★

Example: Construct a histogram of the current population size in each of Idaho's counties.

3. Compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different variables, using statistics appropriate to the shape of the distribution for each measurement variable.★

Example: Compare the histograms of the annual potato yields over the last 25 years for Idaho and Maine using the correct measures of center and spread for the shape of the histograms.

S.ID.B. Summarize, represent, and interpret data on two categorical and quantitative variables.

6. Represent data on two categorical variables on a clustered bar chart and describe how the variables are related. Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.★

Example: Represent the relationship between student effort (on a scale of 1 – 5) and letter grade in a math class with a clustered bar chart and describe the relationship using a relative frequency table.

Teacher Note:

Teacher Note: While this standard is foundational, the type of data and modeling used will extend from foundational into advanced standards.

Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.★

- a. Fit a linear function to data where a scatter plot suggests a linear relationship and use the fitted function to solve problems in the context of the data.★

Teacher Note:

Teachers should emphasize the use of technology when working with this standard. It is more important for students to understand the concept of linear regression and be able to use technology to find the line of best and then interpret it in context than it is to do linear regressions by hand.

S.ID.C. Interpret linear models.

Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.★

Example: Explain why the y -intercept of a linear model relating the volume production of sugar beets to size of farm has no meaning, whereas the y -intercept of a linear model relating the volume production of sugar beets related to minimum temperature does have meaning.

Distinguish between (linear) correlation and causation.★

Making Inferences and Justifying Conclusions – S.IC

S.IC.A. Understand and evaluate random processes underlying statistical studies. Use calculators, spreadsheets, and other technology as appropriate.

1. Understand statistics as a process for making inferences about population parameters based on a random sample from that population.★

Conditional Probability and the Rules of Probability – S.CP

S.CP.A. Understand independence and conditional probability and use them to interpret data from simulations or experiments.

1. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”).★

ADVANCED STANDARDS BY CAREER PATHWAY

These standards are considered pre-requisite standards needed for one or more of the three college pathways. They are more advanced than foundational standards, but will be necessary for success in college-level mathematics courses. Standards in this section that are also listed as the foundational standards are revisited and taught at a deeper level of understanding and application in preparation for college level courses.

This section uses the following course codes for the common general education mathematics courses:

123 = Math in the Modern Society

143 = Precalculus: Algebra

153 = Statistical Reasoning

Number and Quantity (N)

Standard	123	143	153
N.Q.A. Reason quantitatively and use units to solve problems. <ol style="list-style-type: none"> Define appropriate quantities for the purpose of descriptive modeling. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. 	X	X	X
N.CN.A. Perform arithmetic operations with complex numbers. <ol style="list-style-type: none"> Know there is a complex number i such that $i^2 = -1$, and show that every complex number has the form $a + bi$ where a and b are real. Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract and multiply complex numbers. 		X	
N.CN.C. Use complex numbers in polynomial identities and equations. <ol style="list-style-type: none"> Solve quadratic equations with real coefficients that have complex solutions. 		X	

Algebra (A)

Standard	123	143	153
<p>A.SSE.B. Write expressions in equivalent forms to solve problems.</p> <p>3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.</p> <p>b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.</p> <p>Example: <i>A high school player punts a football, and the function $h(t) = -16t^2 + 64t + 2$ represents the height h, in feet, of the football at time t seconds after it is punted. Complete the square in the quadratic expression to find the maximum height of the football.</i></p> <p>4. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1) and use the formula to solve problems.★</p> <p>Example: <i>Calculate mortgage payments.</i></p>	X	X	
<p>A.APR.A. Perform arithmetic operations on polynomials.</p> <p>1. Demonstrate understanding that polynomials form a system analogous to the integers; namely, they are closed under certain operations.</p> <p>a. Perform operations on polynomial expressions (+, -, x, /) and compare the system of polynomials to the system of integers when performing operations.</p> <p><i>Teacher Note: Division with monomials is foundational, but division with polynomials is an Advanced Standard in preparation for Precalculus 1: Algebra.</i></p> <p>b. Factor and/or expand polynomial expressions, identify and combine like terms, and apply the distributive property.</p> <p><i>Teacher Note: Factoring and/or expanding polynomial expressions is an Advanced Standard in preparation for Precalculus 1: Algebra Identifying and combining like terms and applying the distributive property is foundational.</i></p>		X	

Standard	123	143	153
<p>A.APR.B. Understand the relationship between zeros and factors of polynomials.</p> <p>2. Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x-a$ is $p(a)$, so $p(a)=0$ if and only if $x-a$ is a factor of $p(x)$.</p> <p>3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.</p>		X	
<p>A.APR.D. Rewrite rational expressions.</p> <p>7. (+) Demonstrate understanding that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.</p> <p><i>Teacher Note: In preparation for Math 143, students should be able to add, subtract, multiply, and divide rational expressions. The rest of this standard is a college standard introduced in Math 143.</i></p>		X	
<p>A.CED.A. Create equations that describe numbers or relationships.</p> <p>1. Create one-variable equations and inequalities to solve problems, including linear, quadratic, rational, and exponential functions.</p> <p>Example: Four people may be seated at one rectangular table. If two rectangular tables are placed together end-to-end, six people may be seated at the table. If ten tables are placed together end-to-end, how many people can be seated? How many tables are needed for n people?</p> <p>2. Interpret the relationship between two or more quantities</p> <p>a. Define variables to represent the quantities and write equations to show the relationship.</p> <p>b. Use graphs to show a visual representation of the relationship while adhering to appropriate labels and scales.</p> <p>3. Represent constraints using equations or inequalities and interpret solutions as viable or non-viable options in a modeling context.</p>		X	

Standard	123	143	153
4. Represent constraints using systems of equations and/or inequalities and interpret solutions as viable or non-viable options in a modeling context.			
A.REI.A. Understand solving equations as a process of reasoning and explain the reasoning. 2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.		X	
A.REI.B. Solve equations and inequalities in one variable. 3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. a. Solve linear equations and inequalities in one variable involving absolute value. 4. Solve quadratic equations in one variable. a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x-p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula, and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b . <i>Teacher Note: Rather than focusing on students being able to derive the quadratic formula themselves, it is more important for them to make sense of where the quadratic formula comes from. That is, focus on looking for patterns and solutions being in the same format</i>		X	
A.REI.C. Solve systems of equations. 5. Verify that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.		X	

Standard	123	143	153
<p>A.REI.D. Represent and solve equations and inequalities graphically.</p> <p>10. Demonstrate understanding that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane. Show that any point on the graph of an equation in two variables is a solution to the equation.</p> <p>Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.</p>		X	

Functions (F)

Standard	123	143	153
<p>F.IF.B. Interpret functions that arise in applications in terms of the context. Include linear, quadratic, exponential, rational, polynomial, square root and cube root, trigonometric, and logarithmic functions.</p> <p>4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maxima and minima; symmetries; end behavior; and periodicity.</p> <p>5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.</p> <p><i>Teacher Note: Foundational standards primarily emphasize linear functions. Foundational standards also include a less formal exploration of the basic attributes of exponential and quadratic functions. The further reinforcement of these standards (F.IF.B.4 and F.IF.B.5) as prerequisite concepts for the named dual-credit courses would be for continued exploration of exponential and quadratic functions as well as more complex functions such as cubic and logarithmic functions.</i></p>	X	X	X

Geometry (G)

Standard	123	143	153
<p>G.SRT.C. Define trigonometric ratios and solve problems involving right triangles.</p> <p>7. Explain and use the relationship between the sine and cosine of complementary angles.</p> <p><i>Teacher Note: This standard is primarily taught in Precalculus II: Trigonometry. Introducing this concept before Precalculus I: Algebra is helpful preparation.</i></p> <p>8. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.</p> <p><i>Note: This standard is primarily taught in Precalculus II: Trigonometry. Introducing this concept before Precalculus I: Algebra is helpful preparation.</i></p>		X	
<p>G.C.B. Find arc lengths and areas of sectors of circles.</p> <p>5. Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.</p> <p><i>Teacher Note: This standard is primarily taught in Precalculus II: Trigonometry. Introducing this concept before Precalculus I: Algebra is helpful preparation.</i></p>		X	
<p>G.GPE.A. Translate between the geometric description and the equation for a conic section.</p> <p>1. Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.</p> <p>3. (+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.</p> <p>a. Use equations and graphs of conic sections to model real-world problems. ★</p>		X	

Standard	123	143	153
G.GPE.B. Use coordinates to prove simple geometric theorems algebraically. 6. Find the point on a directed line segment between two given points that partitions the segment in a given ratio.		X	
G.GMD.A. Explain volume formulas and use them to solve problems. 2. (+) Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures. 3. Use volume formulas for cylinders, pyramids, cones and spheres to solve problems.	X	X	
G.GMD.B. Visualize relationships between two-dimensional and three-dimensional objects. 4. Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects	X	X	

Statistics and Probability (S)

Standard	123	143	153
S.ID.A. Summarize, represent, and interpret data on a single count or measurement variable. Use calculators, spreadsheets, and other technology as appropriate. 4. Interpret differences in shape, center, and spread in the context of the variables accounting for possible effects of extreme data points (outliers) for measurement variables.★ <i>Example: Describe differences in distributions of annual precipitation over the last 100 years between Boise and Seattle using shape, center, spread, and outliers.</i> 5. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.★			X

Standard	123	143	153
<p>Example: Estimate the percentage of all Idaho elk hunters who successfully filled their tag last year, using the results from Washington County hunters.</p>			
<p>S.ID.B. Summarize, represent, and interpret data on two categorical and quantitative variables.</p> <p>6. Represent data on two categorical variables on a clustered bar chart and describe how the variables are related. Summarize categorical data for two categories in a two-way frequency table. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.</p> <p><i>Teacher Note: Specifically focus on the part, “Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.”</i></p>			X

COLLEGE STANDARDS

These standards are taught at the college level. They appear in dual-credit courses taught in high school and also in college mathematics courses. Standards in this section that are also listed in the foundational and advanced standards are revisited and taught at a deeper level of understanding and application in preparation for college level courses.

This section uses the following course codes for the common general education mathematics courses:

123 = Math in the Modern Society

143 = Precalculus: Algebra

153 = Statistical Reasoning

Number and Quantity (N)

Standard	123	143	153
<p>N.Q.A. Reason quantitatively and use units to solve problems.</p> <p>3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.★</p> <p><i>Teacher Note: In high school students are told the level of accuracy. Students choose a level of accuracy in college level courses.</i></p>			X
<p>N.CN.A. Perform arithmetic operations with complex numbers.</p> <p>1. Know there is a complex number i such that $i^2 = -1$, and show that every complex number has the form $a + bi$ where a and b are real.</p> <p><i>Example: Express the radical $\pm\sqrt{-24}$, using the imaginary unit, i, in simplified form. Expressing the radical using i in simplified form results in the expression $\pm 2i\sqrt{6}$.</i></p> <p><i>Teacher Note: This standards was also marked as a prerequisite for 143. This is still true, but the concepts are reinforced in Precalculus 1: Algebra.</i></p> <p>2. Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.</p>		X	

Standard	123	143	153
<p><i>Teacher Note: This standard was also marked as a prerequisite for 143. This is still true, but the concepts are reinforced in Precalculus 1: Algebra.</i></p> <p>3. N.CN.A.3 (+) Find the conjugate of a complex number; use conjugates to find absolute value and quotients of complex numbers.</p> <p><i>Taught in Precalculus 2: Trigonometry</i></p>			
<p>N.CN.B. Represent complex numbers and their operations on the complex plane.</p> <p>4. (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.</p> <p><i>Taught in Precalculus 2: Trigonometry</i></p> <p>5. (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation.</p> <p><i>Example: $(1 + i\sqrt{3})^3 = 8$ because $(-1 + i\sqrt{3})$ has a radius of 2 and argument 120°.</i></p> <p><i>Taught in Precalculus 2: Trigonometry</i></p> <p>6. (+) Calculate the distance between numbers in the complex plane as the absolute value of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.</p> <p><i>Also taught in Precalculus 2: Trigonometry</i></p>			
<p>N.CN.C. Use complex numbers in polynomial identities and equations.</p> <p>8. (+) Extend polynomial identities to the complex numbers.</p> <p><i>Example: Rewrite $x^2 + 4$ as $(x + 2i)(x - 2i)$.</i></p> <p><i>Also taught in Precalculus 2: Trigonometry</i></p> <p>9. (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.</p>		X	
<p>N.VM.A. Represent and model with vector quantities.</p>			

Standard	123	143	153
<p>1. (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., \mathbf{v}, \mathbf{v}, $\ \mathbf{v}\$, v).</p> <p><i>Taught in Precalculus 2: Trigonometry</i></p> <p>2. (+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.</p> <p><i>Taught in Precalculus 2: Trigonometry</i></p> <p>3. (+) Solve problems involving velocity and other quantities that can be represented by vectors.</p> <p><i>Taught in Precalculus 2: Trigonometry</i></p>			
<p>N.VM.B. Perform operations on vectors.</p> <p>4. (+) Add and subtract vectors.</p> <p>a. (+) Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.</p> <p>b. (+) Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.</p> <p>c. (+) Demonstrate understanding of vector subtraction $\mathbf{v} - \mathbf{w}$ as $\mathbf{v} + (-\mathbf{w})$, where $-\mathbf{w}$ is the additive inverse of \mathbf{w}, with the same magnitude as \mathbf{w} and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.</p> <p><i>Taught in Calculus 2/3 and Linear Algebra</i></p> <p>5. (+) Multiply a vector by a scalar.</p> <p>a. (+) Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$.</p> <p>b. (+) Compute the magnitude of a scalar multiple $c\mathbf{v}$ using $\ c\mathbf{v}\ = c \mathbf{v}$. Compute the direction of $c\mathbf{v}$, knowing that when $c \mathbf{v} \neq 0$, the direction of $c\mathbf{v}$ is either along \mathbf{v} (for $c > 0$) or against \mathbf{v} (for $c < 0$).</p> <p><i>Taught in Calculus 2/3 and Linear Algebra</i></p>			

Standard	123	143	153
<p>N.VM.C. Perform operations on matrices and use matrices in applications.</p> <p>1. (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.</p> <p><i>Taught in Linear Algebra</i></p> <p>2. (+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled. (+) Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.</p> <p><i>Taught in Linear Algebra</i></p> <p>3. (+) Add, subtract, and multiply matrices of appropriate dimensions.</p> <p><i>Taught in Linear Algebra</i></p> <p>4. (+) Demonstrate understanding that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.</p> <p><i>Taught in Linear Algebra</i></p> <p>5. (+) Demonstrate understanding that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.</p> <p><i>Taught in Linear Algebra</i></p> <p>6. (+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.</p> <p><i>Taught in Linear Algebra</i></p> <p>7. Work with 2×2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.</p> <p><i>Taught in Linear Algebra</i></p>			

Algebra (A)

Standard	123	143	153
<p>A.APR.C. Use polynomial identities to solve problems.</p> <p>4. Prove polynomial identities and use the to describe numerical relationships. (Example: The polynomial identity $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples.)</p> <p>Example: The polynomial identity $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples.</p> <p>5. (+) Know and apply the Binomial Theorem for the expansion of $(x+y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.</p> <p><i>Sometimes taught in Precalculus 2: Trigonometry</i></p>			
<p>A.APR.D. Rewrite rational expressions.</p> <p>6. Rewrite simple rational expressions in different forms using inspection, long division, or, for the more complicated examples, a computer algebra system.</p> <p>Example: Write $\frac{a(x)}{b(x)}$ in the form $q(x) + \frac{r(x)}{b(x)}$ where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$.</p> <p>7. Demonstrate understanding that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.</p>		X	

Standard	123	143	153
<p>A.CED.A. Create equations that describe numbers or relationships.</p> <ol style="list-style-type: none"> 1. Create one-variable equations and inequalities to solve problems, including linear, quadratic, rational, and exponential functions. 2. Interpret the relationship between two or more quantities.★ <ol style="list-style-type: none"> a. Define variables to represent the quantities and write equations to show the relationship.★ <p><i>Example: The cost of parking in the parking garage is \$2.00 for the first hour and \$1.00 for every hour after that. Write an equation in terms of x and y that shows the total cost for parking, y, for x hours. Use the equation to calculate the cost for parking in the garage for 5 hours.</i></p> <ol style="list-style-type: none"> b. Use graphs to show a visual representation of the relationship while adhering to appropriate labels and scales.★ <p><i>Example: Using the equation from A.CED.A.2.a, show how the graph of the equation can be used to predict the cost for a specified amount of time.</i></p> <p><i>Teacher Note: This was also marked as a prerequisite for 143. This is still true, but the concepts are reinforced in Precalculus: Algebra.</i></p> 3. Represent constraints using equations or inequalities and interpret solutions as viable or non-viable options in a modeling context.★ <p><i>Teacher Note: This was also marked as a prerequisite for 143. This is still true, but the concepts are reinforced in Precalculus: Algebra.</i></p> <ol style="list-style-type: none"> 4. Represent constraints using systems of equations and/or inequalities and interpret solutions as viable or non-viable options in a modeling context.★ <p><i>Teacher Note: This was also marked as a prerequisite for 143. This is still true, but the concepts are reinforced in Precalculus: Algebra.</i></p>		X	
<p>A.REI.C. Solve systems of equations.</p> <ol style="list-style-type: none"> 7. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. <p><i>Example: Find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.</i></p> <ol style="list-style-type: none"> 8. (+) Represent a system of linear equations as a single matrix equation in a vector variable. <p><i>Taught in Linear Algebra</i></p>		X	

Standard	123	143	153
<p>9. (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3×3 or greater).</p> <p><i>Taught in Linear Algebra</i></p>			
<p>A.REI.D. Represent and solve equations and inequalities graphically.</p> <p>10. Demonstrate understanding that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane. Show that any point on the graph of an equation in two variables is a solution to the equation.</p> <p>11. Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.★</p> <p>Example: Use technology to graph the functions, make tables of values, or find successive approximations.</p>		X	

Functions (F)

Standard	123	143	153
<p>F.IF.A. Understand the concept of a function and use function notation.</p> <p>1. Demonstrate understanding that a function is a correspondence from one set (called the domain) to another set (called the range) that assigns to each element of the domain exactly one element of the range: If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.</p> <p><i>Also taught in Calculus 1 and 2</i></p> <p>3. Demonstrate that a sequence is a functions, sometimes defined recursively, whose domain is a subset of the integers.</p> <p>Example: The Fibonacci sequence is defined recursively by $f(0) = f(1) = 1$, $f(n + 1) = f(n) + f(n - 1)$ for $n \geq 1$.</p> <p><i>Teacher Note: Not all 153 courses include this standard.</i></p>		X	X

Standard	123	143	153
<p>F.IF.B. Interpret functions that arise in applications in terms of the context. Include linear, quadratic, exponential, rational, polynomial, square root and cube root, trigonometric, and logarithmic functions.</p> <p>4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maxima and minima; symmetries; end behavior; and periodicity.★</p> <p><i>Teacher Note: This is addressed as a foundational standard and as an advanced standard. This standard is also addressed at the college level with increasingly complex functions. It is also taught in Precalculus 2: Trigonometry and Calculus.</i></p> <p>6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★</p> <p><i>Also taught in Calculus 1</i></p>		X	
<p>F.IF.C. Analyze functions using different representations.</p> <p>7. Graph functions expressed symbolically and show key features of the graphs, by hand in simple cases and using technology for more complicated cases.★</p> <p>b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.★</p> <p>c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.★</p> <p>d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.★</p> <p>e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.★</p> <p><i>Also taught in Precalculus 2: Trigonometry</i></p> <p>8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.</p>		X	

Standard	123	143	153
<p>a. Use the process of factoring and/or completing the square in quadratic and polynomial functions, where appropriate, to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.</p> <p>Example: Suppose $h(t) = -5t^2 + 10t + 3$ represents the height of a diver above the water (in meters), t seconds after the diver leaves the springboard. What is the maximum height above the water the diver reaches? After how many seconds, t, does the diver hit the water?</p> <p>b. Use the properties of exponents to interpret expressions for exponential functions. Apply to financial situations such as identifying appreciation and depreciation rate for the value of a house or car sometime after its initial purchase.</p> <p>Example: The equation for radioactive decay is $A = A_0 \left(\frac{1}{2}\right)^{\frac{t}{h}}$. When A_0 is the original amount of a radioactive substance, A is the final amount, h is the half-life of the substance, and t is time. Hagerman, Idaho is a hotbed of fossil hunting. The half-life of carbon-14 is about 5730 years. If a fossil that was found in Hagerman contains 54 grams of carbon-14 at time $t = 0$, how much carbon-14 remains at time $t = 17190$ years?</p> <p>10. Given algebraic, numeric and/or graphical representations of functions, recognize the function as polynomial, rational, logarithmic, exponential, or trigonometric.</p> <p><i>Also taught in Precalculus 2: Trigonometry</i></p>			
<p>F.BF.A. Build a function that models a relationship between two quantities.</p> <p>1. Write a function that describes a relationship between two quantities. Functions could include linear, exponential, quadratic, simple rational, radical, logarithmic, and trigonometric.★</p> <p>b. Combine standard function types using arithmetic operations.★</p> <p>Example: Build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.</p> <p>c. (+) Compose functions.★</p>		X	

Standard	123	143	153
<p>Example: If $T(y)$ is the temperature in the atmosphere as a function of height, and $h(t)$ is the height of a weather balloon as a function of time, then $T(h(t))$ is the temperature at the location of the weather balloon as a function of time.</p> <p>Also taught in <i>Precalculus 2: Trigonometry</i></p> <p>2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.★</p> <p>Example: If the U.S. Census Bureau wrote the following recursive equation to represent how they estimate Idaho's population will grow each year after 2019: $P(n) = 1.023 \cdot P(n - 1)$, $P(0) = 1,787,000$. $P(n)$ represents Idaho's population at the end of the n^{th} year in terms of Idaho's population at the end of the $(n - 1)^{\text{th}}$ year, $P(n - 1)$. Predict Idaho's population in 2040.</p> <p>Also taught in <i>Calculus 1 and 2</i></p>			
<p>F.BF.B. Build new functions from existing functions.</p> <p>3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Include, linear, quadratic, exponential, absolute value, simple rational and radical, logarithmic, and trigonometric functions. Utilize technology to experiment with cases and illustrate an explanation of the effects on the graph. Include recognizing even and odd functions from their graphs and algebraic expressions for them.</p> <p>Also taught in <i>Precalculus 2: Trigonometry</i></p> <p>4. Find inverse functions algebraically and graphically.</p> <p>Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse. Include linear and simple polynomial, rational, and exponential functions.</p> <p>Example: $f(x) = 2x^3$ or $f(x) = \frac{x+1}{x-1}$ for $x \neq 1$</p> <p>(+) Verify by composition that one function is the inverse of another.</p>		X	

Standard	123	143	153
<p>(+) Read values of an inverse function from a graph or a table, given that the function has an inverse.</p> <p>(+) Produce an invertible function from a non-invertible function by restricting the domain.</p> <p>5. (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.</p>			
<p>F.LE.A. Construct and compare linear, quadratic, and exponential models and solve problems.</p> <ol style="list-style-type: none"> 1. Distinguish between situations that can be modeled with linear functions and with exponential functions.★ <ol style="list-style-type: none"> a. Demonstrate that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.★ b. Identify situations in which one quantity changes at a constant rate per unit interval relative to another.★ c. Identify situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.★ <p><i>Teacher Note: This standard is also a foundational standard.</i></p> 2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (including reading these from a table).★ <p><i>Teacher Note: This standard is also a foundational standard. However, it takes on a deeper level of focus and understanding in the Calculus sequence.</i></p> 3. Use graphs and tables to demonstrate that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.★ <p><i>Example: Becca's parents are saving for her college education by putting \$3,000/year in a safe deposit box. Becca's grandpa is also saving for her college education by putting \$2,000/year in an IDeal (Idaho college savings) account with an APR of 6.17%. Build tables to show which account has the most money after ten years, and how much more? How many years</i></p>		X	

Standard	123	143	153
<p><i>will it take for the total in her grandpa's account to exceed the total in her parents' safe deposit box</i></p> <p>4. For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.★</p> <p>Example: <i>Mr. Rico has a savings account that has an interest rate of 7% compounded continuously. The amount in the account is calculated using $A = Pe^{rt}$. If Mr. Rico invested \$30,000 on January 1, 2020, when will he have \$100,000 in the account?</i></p>			
<p>F.LE.B. Interpret expressions for functions in terms of the situation they model.</p> <p>5. F.LE.B.5 - Interpret the parameters in a linear or exponential function (of the form $f(x) = b^x + k$) in terms of a context.★</p>		X	
<p>F.TF.A. Extend the domain of trigonometric functions using the unit circle.</p> <p>1. Demonstrate radian measure as the ratio of the arc length subtended by a central angle to the length of the radius of the unit circle.</p> <p>a. Use radian measure to solve problems.</p> <p>Example: <i>You live in New Meadows, Idaho, which is located on the 45th parallel (45° North latitude). Approximately how far will you drive, in miles, to attend the Calgary Stampede? Calgary is located at 51°N latitude, almost due North of New Meadows. (Use $r = 3960$ miles for the radius of the Earth.)</i></p> <p><i>Taught in Precalculus 2: Trigonometry</i></p> <p>2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.</p> <p><i>Taught in Precalculus 2: Trigonometry</i></p> <p>3. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\frac{\pi}{3}$, $\frac{\pi}{4}$, and $\frac{\pi}{6}$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi - x$, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number.</p>			

Standard	123	143	153
<p><i>Taught in Precalculus 2: Trigonometry</i></p> <p>4. (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.</p> <p><i>Taught in Precalculus 2: Trigonometry</i></p>			
<p>F.TF.B. Model periodic phenomena with trigonometric functions.</p> <p>5. F.TF.B.5 - Model periodic phenomena using trigonometric functions with specified amplitude, frequency, and midline.★</p> <p>Example: <i>This past summer you and your friends decided to ride the Ferris wheel at the Idaho State Fair. You wondered how high the highest point on the Ferris wheel was. You asked the operator, and he didn't know, but he told you that the height of the chair was 5 ft off the ground when you got on and the center of the Ferris wheel is 30 ft above that. You checked your phone when you got on and figured out that it took you 12 mins to make one full revolution. Create a model to show your height from the platform at any given time on the Ferris wheel.</i></p> <p><i>Taught in Precalculus 2: Trigonometry</i></p> <p>6. (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.</p> <p><i>Taught in Precalculus 2: Trigonometry</i></p> <p>7. (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.★</p> <p><i>Taught in Precalculus 2: Trigonometry</i></p>			
<p>F.TF.C. Prove and apply trigonometric identities.</p> <p>8. F.TF.C.8 - Relate the Pythagorean Theorem to the unit circle to discover the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use the Pythagorean identity to find the value of a trigonometric function ($\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$) given one trigonometric function ($\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$) and the quadrant of the angle.</p>			

Standard	123	143	153
<p>Example: Suppose that $\cos(\theta) = \frac{2}{5}$ and that θ is in the 4th quadrant. Find the exact value of $\sin(\theta)$ and $\tan(\theta)$.</p> <p><i>Taught in Precalculus 2: Trigonometry</i></p> <p>9. (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.</p> <p><i>Taught in Precalculus 2: Trigonometry and Math Proofs courses</i></p>			

Geometry (G)

Standard	123	143	153
<p>G.SRT.B. Prove theorems involving similarity.</p> <p>4. Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity</p> <p><i>Teacher Note: This standard was deemed unnecessary in high school, and also not covered in college. The use of the Pythagorean theorem is central to foundational mathematics, but the proofs described here are not.</i></p> <p><i>This standard is taught in Axiomatic/Euclidean Geometry at the college level.</i></p>			
<p>G.SRT.D. Apply trigonometry to general triangles.</p> <p>9. (+) Derive the formula $A = \frac{1}{2}ab\sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.</p> <p><i>Taught in Precalculus 2: Trigonometry</i></p> <p>10. (+) Prove the Laws of Sines and Cosines and use them to solve problems.</p> <p><i>Taught in Precalculus 2: Trigonometry and Math Proofs courses</i></p> <p>11. (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems; resultant forces).</p>			

Standard	123	143	153
<i>Taught in Precalculus 2: Trigonometry and College Physics</i>			
<p>G.C.A. Understand and apply theorems about circles.</p> <p>1. Prove that all circles are similar.</p> <p><i>Taught in Axiomatic/Euclidean Geometry</i></p> <p>3. Prove properties of angles for a quadrilateral and other polygons inscribed in a circle, by constructing the inscribed and circumscribed circles of a triangle.</p> <p><i>Taught in Axiomatic/Euclidean Geometry</i></p> <p>4. (+) Construct a tangent line to a circle from a point outside the given circle.</p> <p><i>Taught in Axiomatic/Euclidean Geometry</i></p>			
<p>G.GPE.A. Translate between the geometric description and the equation for a conic section.</p> <p>2. Derive the equation of a parabola given a focus and directrix.</p> <p><i>Taught in Calculus 2</i></p> <p>3. (+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.</p> <p><i>Taught in Calculus 2 and 3</i></p>			
<p>G.GMD.A. Explain volume formulas and use them to solve problems.</p> <p>2. (+) Give an informal argument using Cavalieri’s principle for the formulas for the volume of a sphere and other solid figures.</p> <p><i>Taught in Axiomatic/Euclidean Geometry</i></p> <p>3. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.★</p> <p>Example: <i>The tank at the top of the Meridian Water Tower is roughly spherical. If the diameter of the sphere is 50.35 feet, approximately how much water can the tank hold?</i></p>			

Standard	123	143	153
<i>Teacher Note: This standard is first introduced as a foundational standard and is developed further in the advanced standards. It is then revisited in Axiomatic/Euclidean Geometry.</i>			
<p>G.GMD.B. Visualize relationships between two-dimensional and three-dimensional objects.</p> <p>4. Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.</p> <p><i>Taught in Calculus 2 and 3</i></p>			
<p>G.MG.A. Apply geometric concepts in modeling situations.</p> <p>4. Use dimensional analysis for unit conversions to confirm that expressions and equations make sense.★</p> <p><i>Teacher Note: This standard also appears as a foundational standard as the complexity of applications of this standard increases.</i></p>		X	

Statistics and Probability (S)

Standard	123	143	153
<p>S.ID.B. Summarize, represent, and interpret data on two categorical and quantitative variables.</p> <p>7. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.★</p> <p>b. Use functions fitted to data, focusing on quadratic and exponential models, or choose a function suggested by the context. Utilize technology where appropriate.★</p> <p>Example: Use technology to fit a function of the relationship between the board-feet (measured in volume) of trees and the diameter of the trunks of the trees.</p> <p>c. Informally assess the fit of a function by plotting and analyzing residuals.★</p>			X

Standard	123	143	153
<p>S.ID.C. Interpret linear models.</p> <p>9. Compute (using technology) and interpret the linear correlation coefficient.★</p> <p><i>Example: Find the correlation coefficient between the number of hours firefighters sleep each night and the length of fireline they construct each day. Use the correlation coefficient to explain whether sleep is important.</i></p>			X
<p>S.IC.A. Understand and evaluate random processes underlying statistical studies. Use calculators, spreadsheets, and other technology as appropriate.</p> <p>2. Decide if a specified model is consistent with results from a given data-generating process (e.g., using simulation or validation with given data).★</p> <p><i>Example: A model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?</i></p>			X
<p>S.IC.B. Make inferences and justify conclusions from sample surveys, experiments, and observational studies.</p> <p>3. Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.★</p> <p>4. Use data from a sample survey to estimate a population mean or proportion and a margin of error.★</p> <p>5. Use data from a randomized and controlled experiment to compare two treatments; use margins of error to decide if differences between treatments are significant.★</p> <p>6. Evaluate reports of statistical information based on data.★</p> <p><i>Example: Students may analyze and critique different reports from media, business, and government sources.</i></p>			X
<p>S.CP.A. Understand independence and conditional probability and use them to interpret data from simulations or experiments.</p>			X

Standard	123	143	153
<p>2. Demonstrate understanding that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.★</p> <p>3. Understand the conditional probability of A given B as $\frac{P(A \cap B)}{P(B)}$, and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.★</p> <p><i>Taught in Statistical Methods and Probability and Statistics courses</i></p> <p>4. Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.★</p> <p>Example: <i>Collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.</i></p> <p>5. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.★</p> <p>Example: <i>Compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.</i></p> <p><i>Also taught in Statistical Methods and Probability and Statistics courses</i></p>			
<p>S.CP.B. Use the rules of probability to compute probabilities of compound events in a uniform probability model.</p> <p>6. Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.★</p> <p><i>Taught in Statistical Methods and Probability and Statistics courses</i></p> <p>7. Apply the Addition Rule, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$, and interpret the answer in terms of the model.★</p>			X

Standard	123	143	153
<p><i>Also taught in Statistical Methods and Probability and Statistics courses</i></p> <p>8. (+) Apply the general Multiplication Rule in a uniform probability model $P(A \cap B) = P(A)P(B) = P(B)P(A B)$, and interpret the answer in terms of the model.★</p> <p><i>Also taught in Statistical Methods and Probability and Statistics courses</i></p> <p>9. (+) Use permutations and combinations to compute probabilities of compound events and solve problems.★</p> <p><i>Also taught in Statistical Methods and Probability and Statistics courses</i></p>			
<p>S.MD.A. Calculate expected values and use them to solve problems.</p> <p>1. (+) Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.★</p> <p><i>Taught in Statistical Methods and Probability and Statistics courses</i></p> <p>2. (+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution of the variable.★</p> <p><i>Also taught in Statistical Methods and Probability and Statistics courses</i></p> <p>3. (+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value.★</p> <p>Example: Find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.</p> <p><i>Taught in Statistical Methods and Probability and Statistics courses</i></p> <p>4. (+) Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value.★</p> <p>Example: Find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets</p>			X

Standard	123	143	153
<p><i>per household. How many TV sets would you expect to find in 100 randomly selected households?</i></p> <p><i>Taught in Statistical Methods and Probability and Statistics courses</i></p>			
<p>S.MD.B. Use probability to evaluate outcomes of decisions.</p> <p>5. (+) Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.★</p> <p>a. Find the expected payoff for a game of chance.★</p> <p>Example: Find the expected winnings from a state lottery ticket or a game at a fast-food restaurant.</p> <p>b. Evaluate and compare strategies on the basis of expected values.★</p> <p>Example: Compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.</p> <p><i>Also taught in Statistical Methods and Probability and Statistics courses</i></p> <p>6. (+) Use probabilities to make objective decisions.★</p> <p>Example: The Idaho Department of Transportation classifies highways for overweight loads based on the probability of bridges on a highway failing under given vehicle weights.</p> <p><i>Also taught in Statistical Methods and Probability and Statistics courses</i></p> <p>7. (+) Analyze decisions and strategies using probability concepts.★</p> <p>Example: Product testing, medical testing, or pulling a hockey or soccer goalie at the end of a game and replacing the goalie with an extra player.</p> <p><i>Also taught in Statistical Methods and Probability and Statistics courses</i></p>			X

REFERENCES

- Aughinbaugh, A. (2012). The effects of high school math curriculum on college attendance: Evidence from the NLSY97. *Economics of Education Review*, 31 (6), 861-870. Retrieved from: <https://www.sciencedirect.com/science/article/abs/pii/S0272775712000726>
- Jia, N. (2021). Do stricter high school math requirements raise college STEM attainment? *Economics of Education Review*, 83. Retrieved from: <https://www.sciencedirect.com/science/article/abs/pii/S0272775721000595>
- National Council Teachers of Mathematics. (2014). *Principles to Actions: Ensuring Mathematical Success for All*. NCTM.
- Son, J.Y. & Stigler, J. (2023) Don't Force a False Choice between Algebra and Data Science. EdSource. Retrieved from: <https://edsources.org/2023/dont-force-a-false-choice-between-algebra-and-data-science/684817>
- Sparks, S.D. (2018) Calculus is the Peak of High School Math. Maybe It's Time to Change That. *Education Week*, 37(32). Retrieved from: www.edweek.org/ew/articles/2018/05/23/calculus-is-the-peak-of-high-school.html?cmp=eml-enl-cm-news2-rm&M=58504831&U=1121672
- University of Texas at Austin Charles A. Dana Center. (2020). Launch Years: A New Vision for the Transition from High School to Postsecondary Mathematics. Retrieved from: <https://www.utdanacenter.org/our-work/k-12-education/launch-years-initiative/launch-years-initiative-resources-and-reports>
- Zelkowski, J. (2011) Defining the Intensity of High School Mathematics: Distinguishing the Difference Between College-Ready and College-Eligible Students. *American Secondary Education*, 39(2), 27-54. Retrieved from: <https://www.jstor.org/stable/23100400>

ADDITIONAL RECOMMENDED READING

- Boaler, J. (2022). *Mathematical Mindsets*. Jossey-Bass.
- Leinwand, S. & Milou, E. (2021). *Invigorating High School Math*. Heinemann.
- National Council of Teachers of Mathematics. (2018). *Catalyzing Change in High School Mathematics*. NCTM.
- National Council of Teachers of Mathematics. (2016). *Mathematical Modeling and Modeling Mathematics*. NCTM.
- National Council of Supervisors of Mathematics. Closing the Opportunity Gap: A Call for Detracking Mathematics. Position paper retrieved from: <https://www.mathedleadership.org/docs/resources/positionpapers/NCSMPositionPaper19.pdf>

APPENDIX A: 9-12 STANDARDS OVERVIEW CHART

Key: F=Foundational, A=Advanced, C=College

Number and Quantity			
Standard	F	A	C
N.RN.A.1	x		
N.RN.A.2	x		
N.RN.B.3	x		
N.Q.A.1	x		
N.Q.A.2	x	x	
N.Q.A.3		x	x
N.CN.A.1		x	x
N.CN.A.2		x	x
N.CN.A.3			x
N.CN.B.4			x
N.CN.B.5			x
N.CN.B.6			x
N.CN.C.7		x	
N.CN.C.8			x
N.CN.C.9			x
N.VM.A.1			x
N.VM.A.2			x
N.VM.A.3			x
N.VM.B.4			x
N.VM.B.4a			x
N.VM.B.4b			x
N.VM.B.4c			x
N.VM.B.5			x
N.VM.B.5a			x
N.VM.B.5b			x
N.VM.C.6			x
N.VM.C.7			x
N.VM.C.8			x

Number and Quantity			
Standard	F	A	C
N.VM.C.9			x
N.VM.C.10			x
N.VM.C.11			x
N.VM.C.12			x

Algebra			
Standard	F	A	C
A.SSE.A.1	x		
A.SSE.A.1a	x		
A.SSE.A.1b	x		
A.SSE.A.2	x		
A.SSE.B.3	x		
A.SSE.B.3a	x		
A.SSE.B.3b		x	
A.SSE.B.3c	x		
A.SSE.B.4		x	
A.APR.A.1	x		
A.APR.A.1a	x	x	
A.APR.A.1b	x	x	
A.APR.B.2		x	
A.APR.B.3		x	
A.APR.C.4			x
A.APR.C.5			x
A.APR.D.6			x
A.APR.D.7		x	x
A.CED.A.1	x	x	x
A.CED.A.2	x	x	x
A.CED.A.2a	x	x	x

Algebra			
Standard	F	A	C
A.CED.A.2b	x	x	x
A.CED.A.3	x	x	x
A.CED.A.4	x	x	x
A.CED.A.5	x		
A.REI.A.1	x		
A.REI.A.2		x	
A.REI.B.3	x		
A.REI.B.3a		x	
A.REI.B.4		x	
A.REI.B.4a		x	
A.REI.B.4b		x	
A.REI.C.5		x	
A.REI.C.6	x		
A.REI.C.7			x
A.REI.C.8			x
A.REI.C.9			x
A.REI.D.10	x	x	x
A.REI.D.11			x
A.REI.D.12		x	

Functions			
Standard	F	A	C
F.IF.A.1	x		x
F.IF.A.2	x		
F.IF.A.3			x
F.IF.B.4	x	x	x
F.IF.B.5	x	x	
F.IF.B.6	x		x

Functions			
Standard	F	A	C
F.IF.C.7	x		x
F.IF.C.7a	x		
F.IF.C.7b			x
F.IF.C.7c			x
F.IF.C.7d			x
F.IF.C.7e			x
F.IF.C.8	x		x
F.IF.C.8a			x
F.IF.C.8b			x
F.IF.C.9	x		
F.IF.C.10			x
F.BF.A.1	x		x
F.BF.A.1a	x		
F.BF.A.1b			x
F.BF.A.1c			x
F.BF.A.2			x
F.BF.B.3			x
F.BF.B.4			x
F.BF.B.4a			x
F.BF.B.4b			x
F.BF.B.4c			x
F.BF.B.4d			x
F.BF.B.5			x
F.LE.A.1	x		x
F.LE.A.1a	x		x
F.LE.A.1b	x		x
F.LE.A.1c	x		x
F.LE.A.2	x		x

Functions			
Standard	F	A	C
F.LE.A.3			x
F.LE.A.4			x
F.LE.B.5	x		x
F.TF.A.1			x
F.TF.A.1a			x
F.TF.A.2			x
F.TF.A.3			x
F.TF.A.4			x
F.TF.B.5			x
F.TF.B.6			x
F.TF.B.7			x
F.TF.C.8			x
F.TF.C.9			x

Geometry			
Standard	F	A	C
G.CO.A.1	x		
G.CO.A.2	x		
G.CO.A.3	x		
G.CO.A.4	x		
G.CO.A.5	x		
G.CO.A.6	x		
G.CO.B.7	x		
G.CO.B.8	x		
G.CO.B.9	x		
G.CO.C.10	x		
G.CO.C.11	x		

Geometry			
Standard	F	A	C
G.CO.C.12	x		
G.CO.C.12a	x		
G.CO.D.13	x		
G.CO.D.14	x		
G.SRT.A.1	x		
G.SRT.A.1a	x		
G.SRT.A.1b	x		
G.SRT.A.2	x		
G.SRT.A.3	x		
G.SRT.B.4			x
G.SRT.B.5	x		
G.SRT.C.6	x		
G.SRT.C.7		x	
G.SRT.C.8		x	
G.SRT.D.9			x
G.SRT.D.10			x
G.SRT.D.11			x
G.C.A.1			x
G.C.A.2	x		
G.C.A.3			x
G.C.A.4			x
G.C.B.5		x	
G.GPE.A.1		x	
G.GPE.A.2			x
G.GPE.A.3		x	x
G.GPE.A.3a		x	
G.GPE.B.4	x		

Geometry			
Standard	F	A	C
G.GPE.B.5	x		
G.GPE.B.6	x	x	
G.GPE.B.7	x		
G.GMD.A.1	x		
G.GMD.A.2		x	x
G.GMD.A.3	x	x	x
G.GMD.B.4		x	x
G.MG.A.1	x		
G.MG.A.2	x		
G.MG.A.3	x		
G.MG.A.4	x		x

Statistics			
Standard	F	A	C
S.ID.A.1	x		
S.ID.A.2	x		
S.ID.A.3	x		
S.ID.A.4		x	
S.ID.A.5		x	
S.ID.B.6	x	x	
S.ID.B.7	x		x
S.ID.B.7a	x		
S.ID.B.7b			x
S.ID.B.7c			x
S.ID.C.8	x		
S.ID.C.9			x
S.ID.C.10	x		

Statistics			
Standard	F	A	C
S.IC.A.1	x		
S.IC.A.2			x
S.IC.B.3			x
S.IC.B.4			x
S.IC.B.5			x
S.IC.B.6			x
S.CP.A.1	x		
S.CP.A.2			x
S.CP.A.3			x
S.CP.A.4			x
S.CP.A.5			x
S.CP.B.6			x
S.CP.B.7			x
S.CP.B.8			x
S.CP.B.9			x
S.MD.A.1			x
S.MD.A.2			x
S.MD.A.3			x
S.MD.A.4			x
S.MD.B.5			x
S.MD.B.5a			x
S.MD.B.5b			x
S.MD.B.6			x
S.MD.B.7			x

APPENDIX B: STANDARDS TO ISAT ALIGNMENT

This section shows how the Grade 11 Idaho Standards for Achievement (ISAT) Assessment Claims and Targets align to the different categories of high school mathematics standards. Half of the ISAT is scored based on claims 2, 3, and 4, and these claims are centered on mathematical practices, not just mathematical content ([see ISAT blueprints for details](#)). Instruction and planning should not be limited to ISAT Claim 1, Concepts and Procedures. For instructional priority planning, please primarily reference the [Idaho Content Standards for Mathematics](#).

Note that it is possible to see standards in claims 2 through 4 that are not listed in this table. The tables in this section were created based on the [Smarter Balanced Content Specifications](#) document for mathematics (2015). (retrieved from: <http://portal.smarterbalanced.org/library/en/Mathematics-content-specifications.pdf>)

Grade 11 ISAT Target Summary for Claim 1

Claim 1: Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.

Targets marked with a are ISAT major targets for 11th grade. The other targets are supporting targets. Standards marked with a ★ indicate a Modeling Standard.

Target	Foundational	Advanced	College
Target A - Extend the properties of exponents to rational exponents; Use properties of rational and irrational numbers; Reason quantitatively and use units to solve problems	N.RN.A.1, N.RN.A.2		
Target B - Use properties of rational and irrational numbers	N.RN.B.3		
Target C - Reason quantitatively and use units to solve problems	N.Q.A.1		
<input type="checkbox"/> Target D - Interpret the structure of expressions	A.SSE.A.2		

Target	Foundational	Advanced	College
<input type="checkbox"/> Target E - Write expressions in equivalent forms to solve problems	A.SSE.B.3 A.SSE.B.3a★ A.SSE.B.3c★	A.SSE.B.3b★	
<input type="checkbox"/> Target F - Perform arithmetic operations on polynomials	A.APR.A.1		
Target G - Create equations that describe numbers or relationships	A.CED.A.1★ A.CED.A.2★	A.CED.A.1★ A.CED.A.2★	A.CED.A.1★, A.CED.A.2★
<input type="checkbox"/> Target H - Understand solving equations as a process of reasoning and explain the reasoning		A.REI.A.2	
<input type="checkbox"/> Target I - Solve equations and inequalities in one variable	A.REI.B.3	A.REI.B.4, A.REI.B.4a A.REI.B.4b	
<input type="checkbox"/> Target J - Represent and solve equations and inequalities graphically		A.REI.D.12	A.REI.D.10, A.REI.D.11★,
<input type="checkbox"/> Target K - Understand the concept of a function and use function notation	F.IF.A.1 F.IF.A.3		F.IF.A.1
<input type="checkbox"/> Target L - Interpret functions that arise in applications in terms of a context	F.IF.B.4★ F.IF.B.5★ F.IF.B.6★	F.IF.B.4★ F.IF.B.5★	F.IF.B.4★ F.IF.B.6★
<input type="checkbox"/> Target M - Analyze functions using different representations	F.IF.C.9		F.IF.C.7, F.IF.C.7a★ F.IF.C.7b★ F.IF.C.7c★ F.IF.C.7d★ F.IF.C.7e★ F.IF.C.8a F.IF.C.8b
<input type="checkbox"/> Target N - Build a function that models a relationship between two quantities	F.BF.A.1★ F.BF.A.1a★		F.BF.A.1★, F.BF.A.1b★, F.BF.A.1c★, F.BF.A.2★

Target	Foundational	Advanced	College
Target O - Define trigonometric ratios and solve problems involving right triangles	G.SRT.C.6★	G.SRT.C.7 G.SRT.C.8	
Target P - Summarize, represent, and interpret data on a single count or measurement variable	S.ID.A.1★ S.ID.A.2★ S.ID.A.3★		

Grade 11 ISAT Target Summary for Claims 2 through 4

The performance tasks are primarily used to assess Claims 2 through 4. Note that half of the ISAT is scored based on claims 2, 3, and 4, and these claims are centered on mathematical practices, not just mathematical content ([see ISAT blueprints for details](#)). Since the ISAT outlines clusters (for example, A.SSE.A) and standards (for example, A.SSE.A.1) for claims 2-4, this table breaks clusters into their constituent standards when they are in different categories. A cluster by itself implies that all its constituent standards are included. Performance tasks are written to assess the standards and clusters shown in the table below.

Claim 2 Problem Solving & Claim 4 Modeling and Data Analysis

Claim 2: Students can solve a range of complex, well-posed problems in pure and applied mathematics, making productive use of knowledge and problem-solving strategies.

Claim 4: Students can analyze complex, real-world scenarios and can construct and use mathematical models to interpret and solve problems.

Foundational	Advanced	College
N.Q.A.1 N.Q.A.2	N.Q.A.2 N.Q.A.3	N.Q.A.3
A.SSE.A		
A.SSE.B		
A.CED.A	HS.A.CED.A.1 HS.A.CED.A.2 HS.A.CED.A.2a HS.A.CED.A.2b HS.A.CED.A.3 HS.A.CED.A.4	HS.A.CED.A.2 HS.A.CED.A.2a HS.A.CED.A.2b HS.A.CED.A.3 HS.A.CED.A.4
A.REI.A.1	A.REI.A.2	
A.REI.B.3	A.REI.B.3a	

Foundational	Advanced	College
	A.REI.B.4 A.REI.B.4a A.REI.B.4b	
A.REI.C.6	A.REI.C.5	A.REI.C.7 A.REI.C.8 A.REI.C.9
	A.REI.D.12	A.REI.D.10 A.REI.D.11
HS.F.IF.A.1 HS.F.IF.A.2		HS.F.IF.A.1 HS.F.IF.A.3
F.IF.B	F.IF.B.4 F.IF.B.5	F.IF.B.4 F.IF.B.6
F.IF.C.7 F.IF.C.7a F.IF.C.8 F.IF.C.9		F.IF.C.7 F.IF.C.7a F.IF.C.7b F.IF.C.7c F.IF.C.7d F.IF.C.7e F.IF.C.8 F.IF.C.8a F.IF.C.8b F.IF.C.10
F.BF.A.1 F.BF.A.1a		F.BF.A.1 F.BF.A.1b F.Bf.A.1c F.BF.A.2
F.LE.A.1 F.LE.A.1a F.LE.A.1b. F.LE.A.1c		F.LE.A.1 F.LE.A.1a F.LE.A.1b. F.LE.A.1c F.LE.A.2 F.LE.A.3 F.LE.A.4
F.LE.B.5		F.LE.B.5
		F.TF.B.5
G.SRT.C.6	G.SRT.C.7	

Foundational	Advanced	College
	G.SRT.C.8	
G.GMD.A.3	G.GMD.A.3	G.GMD.A.3
G.MG.A.1 G.MG.A.2 G.MG.A.3 G.MG.A.4		G.MG.A.4
S.ID.A.1 S.ID.A.2 S.ID.A.3	S.ID.A.4 S.ID.A.5	
S.ID.B.6 S.ID.B.7 S.ID.B.7a	S.ID.B.6	S.ID.B.7 S.ID.B.7b S.ID.B.7c
S.ID.C.8 S.ID.C.10		S.ID.C.9
S.IC.A.1		
		S.IC.B.3 S.IC.B.4 S.IC.B.5 S.IC.B.6
S.CP.A.1		S.CP.A.2 S.CP.A.3 S.CP.A.4 S.CP.A.5

Claim 3 Communicating Reasoning

Claim 3: Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others.

Foundational	Advanced	College
N.RN.A		
N.RN.B		
A.SSE.A.2		
A.APR.A.1		
	A.APR.B	
	A.APR.C	

Foundational	Advanced	College
	A.APR.D.6	
A.REI.A.1	A.REI.A.2	
A.REI.C.6	A.REI.C.5	A.REI.C.7 A.REI.C.8 A.REI.C.9
	A.REI.D.12	A.REI.D.10 A.REI.D.11
F.IF.A.1		F.IF.A.1
F.IF.B.5	F.IF.B.5	
F.IF.C.9		
		F.BF.B.3 F.BF.B.4
		F.TF.A.1 F.TF.A.2
		F.TF.C.8
G.CO.A		
G.CO.B		
G.CO.C10 G.CO.C.11 G.CO.C.12		
G.SRT.A		
G.SRT.B.5		G.SRT.B.4